

# Dynamic Tonality: Extending the Framework of Tonality into the 21<sup>st</sup> Century

Jim Plamondon  
6911 Thistle Hill Way  
Austin, TX 78754 USA  
[jim@plamondon.net](mailto:jim@plamondon.net)

Andrew Milne  
Department of Music  
University of Jyväskylä  
Finland  
[andy@tonalcentre.org](mailto:andy@tonalcentre.org)

William Sethares,  
Department of Electrical and  
Computer Engineering  
University of Wisconsin-Madison  
Madison, WI 53706 USA  
[seth@ece.wisc.edu](mailto:seth@ece.wisc.edu)

## ABSTRACT

This paper describes Dynamic Tonality, a system of real-time alterations to tuning and timbre that extends the framework of tonality to include new structural resources such as polyphonic tuning bends, tuning progressions, and temperament modulations. These new resources could prepare art music for the 21<sup>st</sup> Century.

## Author Keywords

Music, tonality, music control interfaces, alternative controllers, tuning, temperament, non-Western music, alternative tunings, microtonality.

## CRISIS IN ART MUSIC

Many leading academics believe that art music is in crisis. For example, the Call for Papers for the 2008 annual conference of the College Music Society [4] states

*“Some...suggest that what is happening in music today is not at all dissimilar to a global warming crisis for our field. If a certain desperation is detected in this call, it is meant only to break through the encrusted proprieties of academic pretension. Ideas that may have served us well in the past [may] now hold us back. The Program Committee invites new, thought-provoking, even revolutionary ideas.”*

This paper suggests that this self-described crisis can be addressed, at least in part, through the exploration of novel musical resources—including polyphonic tuning bends, tuning modulations, and temperament modulations—collectively called Dynamic Tonality [14]. By expanding the framework of tonality to include these new resources, Dynamic Tonality may help to prepare art music for the 21<sup>st</sup> Century.

## INVARIANCE AND ISOMORPHISM

Dynamic Tonality is enabled exposure of *invariance* (i.e., consistency under transformation) in musical data through *isomorphism* (i.e., consistency of mapping).

Isomorphic button-fields, discussed below, expose the musical properties *tuning invariance* and *transpositional invariance*.

## Transpositional Invariance

It is well-known that the familiar interval patterns of music theory<sup>1</sup>—such as the diatonic scale, the major triad, the V–I cadence, the ii–V–I chord progression, etc.—have the property of *transpositional invariance*: they do not change when transposed to a different key.

## Tuning Invariance

A second invariant property of music has only recently (2007) been recognized: *tuning invariance* [12].

A detailed discussion of temperaments and tuning theory is beyond the scope of this paper (see Wikipedia and its references).

In brief, a two-dimensional temperament is defined by two intervals known as its *period* and *generator*—its “two dimensions,” collectively called its *generators*—and a *comma sequence* [16]. A temperament describes an association between a tuning’s notes and selected ratios of small whole number (as found in the Harmonic Series and Just Intonation) [12]. A temperament’s *tuning continuum* is bounded by the tuning points at which some of a given scale’s steps shrink to unison [12].

One such temperament is the *syntonic temperament* [12]. Its period is the octave (P8) and its generator is the tempered perfect fifth (P5). The first comma in its comma sequence is the syntonic comma, after which the temperament is named. All of the other familiar tonal intervals—the minor second, the major third, the augmented sixth, etc.—are generated by combinations of tempered perfect fifths and octaves, as per rules encapsulated in the temperament’s generators and comma sequence. For example, having the syntonic comma come first in the temperament’s comma sequence defines the

---

<sup>1</sup> Unless otherwise specified herein, “music theory” is used to denote the theory of tonal music in Europe’s Common Practice Era (c. 1600-1900).

major third (M3) to be equal in width to four P5's, minus two P8's (*i.e.*,  $M3 = [(4 * P5) + (-2 * P8)]$ ).

Holding the octave constant at 1200 cents and varying the width of the P5 in a smooth continuum from a low of 686 cents to a high of 720 cents produces the *syntonic tuning continuum* shown in Figure 1 below [12].

In Figure 1, “N-TET” stands for “N-tone equal temperament.” The tuning is equivalent to standard 12-tone equal temperament tuning—12-TET—when P5 = 700 cents.

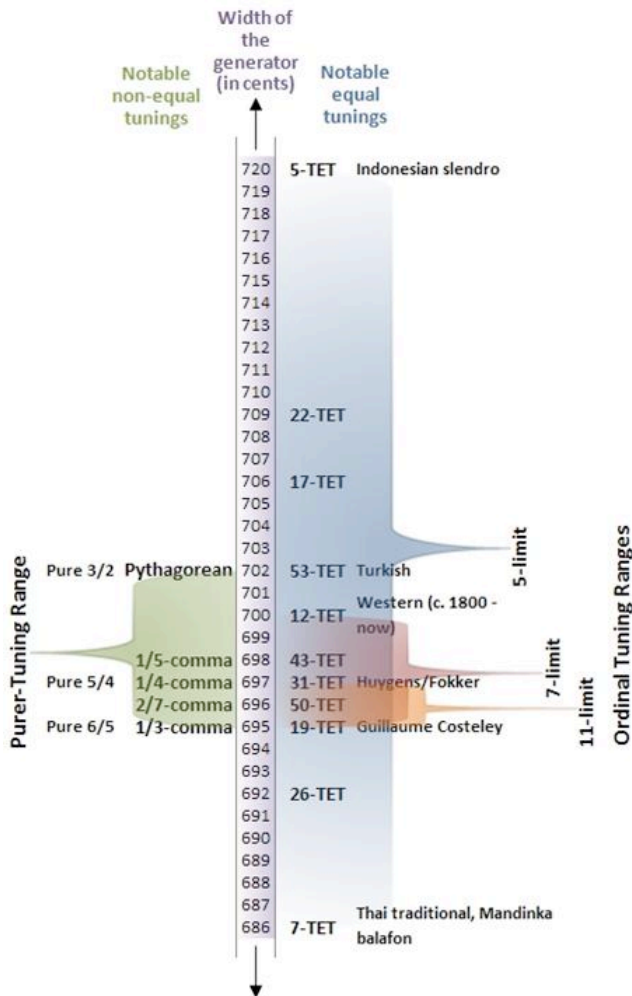


Figure 1: The syntonic tuning continuum.

The syntonic tuning continuum includes many tunings used by Western culture (both today and in the past) and by the indigenous music of many non-Western cultures. Pythagorean tuning ( $P5 \approx 702$ ) and  $\_$ -comma meantone tuning ( $P5 \approx 696.6$ ) have a long history in European music [1]. Ancient Chinese bronze bells may also have been tuned in  $\_$ -comma meantone [3]. Some Arabic music has been described as using extended Pythagorean tuning,

although notated in 24-TET [7]. 17-TET, 19-TET, and especially 31-TET have received attention from Western music theorists [2].

Things get even more interesting at the ends of the tuning continuum. As the width of P5 rises towards 720 cents, the width of the minor second shrinks to zero, leaving a 5-TET scale that is closely related to the Indonesian *slendro* scale [17]. As the width of P5 falls towards 686 cents, the minor second increases in width to match the major second, producing a 7-TET scale similar to that used in the traditional music of Thailand [17] and Mandinka Africa [9].

Unfortunately, traditional musical instruments do not expose music's invariant properties. To expose them, we need a new instrument: an electronic and isomorphic button-field.

### THE ISOMORPHIC BUTTON-FIELD

Transpositional invariance is exposed by isomorphic note-layouts [10], and so is tuning invariance [12].

A *button-field* is a two-dimensional geometric arrangement of note-controlling buttons. A *note-layout* is a mapping of notes to a button-field. The word *isomorphic* comes from the Greek roots “iso” (equal) and “morph” (shape), and we use it to mean “same shape.”

On an *isomorphic note-layout*, pressing any two button-field buttons that have a given geometric relationship to each other sounds a given musical interval. For example, in Figure 2 below, pressing any two buttons that are horizontally adjacent sounds the interval of a major second. The shape of the imaginary line connecting the centers of those two buttons is the *button-field shape* of the major second. If a given isomorphic note-layout and a given temperament have the same generators, then the note-layout will expose the transpositional and tuning invariances of that temperament [12].

One such isomorphic note-layout, shown in Figure 2 below, was described by Kaspar Wicki [6] in 1896. It has the same generators as the syntonic temperament, and therefore exposes that temperament's invariant properties.

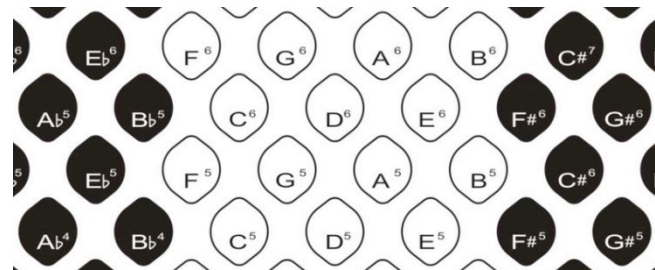
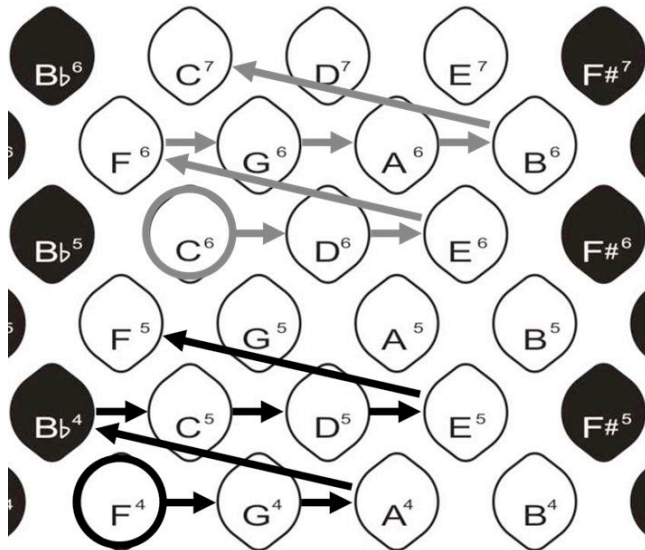


Figure 2: A button-field optimized for use with the isomorphic Wicki note-layout.

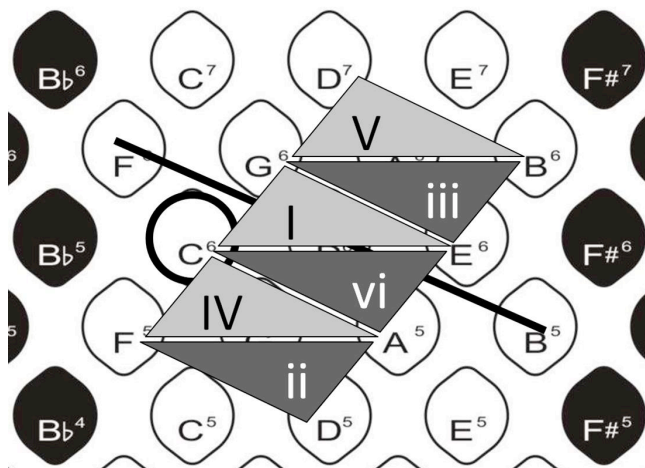
The Wicki note-layout has been used in squeeze-box instruments such as bandoneons, concertinas, and bayans. The historical use of such instruments to competently perform a wide variety of complex and challenging chromatic music shows that the Wicki note-layout is a practical music-control interface.



**Figure 3: Isomorphism of C Major scale (grey lines) and F Major scale (black lines).**

Examples of the button-field’s transpositional invariance are shown in Figures 3 and 4.

Figure 3 above shows the major scale in C (grey arrows) and F (black arrows). The scale’s button-field shape is the same in both keys, and indeed in every key.



**Figure 4: Diatonic tertian triads of C Major.**

The button-field shape of each kind of triad in root position—major (IV, I, & V), minor (ii, vi, & iii), and diminished (vii)—directly reflects the shape of the “stack

of intervals” from which it is constructed. The button-field shapes of these triads, and the geometric relationships among the triads, are invariant within a key and across all keys.

Figure 4 shows the diatonic tertian triads of C Major—that is, the triads built by stacking diatonic thirds on top of the root of each degree of the C Major scale.

All along the syntonic tuning continuum pitches change, interval widths change, and even enharmonic equivalencies change, but *the geometric relationships among tonal intervals on an isomorphic keyboard are invariant* [12]. For example, on the Wicki note-layout, the C major pentatonic scale is played as “C D E G A” all along the tuning continuum. Whether the current tuning has 5 notes per octave, 12 notes per octave, or even 31 notes per octave, the shape of any given musical structure is invariant on the Wicki note-layout.

From the performer’s perspective, the combination of tuning invariance and transpositional invariance can be described as *fingering invariance*, meaning that once a performer has learned to play any given sequence or combination of musical intervals in any one octave, key, and tuning, then the performer can play them in any other octave, key, and tuning with the same pattern of finger-movements (button-field edge conditions aside). Fingering invariance facilitates the exploration of alternative and microtonal tunings, opening new creative frontiers for art music in the 21<sup>st</sup> Century.

### OTHER TEMPERAMENTS

The syntonic temperament’s tuning continuum includes today’s standard 12-TET tuning, many previously-common Western tunings, and many non-Western tunings, as discussed above. Nonetheless, there are other temperaments—e.g., hanson, porcupine, magic, etc. [5]—that are also compatible with Dynamic Tonality, as introduced below.

For ease of discussion, this paper assumes the use of the syntonic temperament unless otherwise specified.

### DYNAMIC TONALITY

Dynamic Tonality consists of two related ideas: Dynamic Tuning and Dynamic Timbres.

#### Dynamic Tuning

Tuning invariance enables real-time changes in tuning along its tuning continuum while retaining consistent fingering. We call such a smooth, real-time, polyphonic tuning change, driven by widening or narrowing the tempered P5, a *polyphonic tuning bend*.

In such a bend, the notes flatter than the tonic (around a circle of fifths) change pitch in one direction while those sharper than the tonic change pitch in the other direction,

with each note's rate of change being directly proportional to its distance from the tonic along a line of P5's.

### Enhancing Expressiveness

Altering the width of the P5 in real time allows a performer to emulate the dynamic tuning of string and wind players who prefer Pythagorean (or higher) tunings when playing expressive melodies,  $\_$ -comma meantone when playing consonant harmonies, and 12-TET when playing with fixed pitch instruments such as the piano [18].

### Tuning Progressions

A sequence of tuning bends can form a *tuning progression*. One could start by playing C Major's tonic triad in 12-TET ( $P5 = 700$ ), then sliding the width of the P5 generator up to 5-TET ( $P5 \approx 720$ ). The pitch of C would not change at all (because it is the tonic). However, the pitch of G would rise by 20 cents (because it's one P5 away from C, and all P5's have been widened by 20 cents), and the width of E would rise by 80 cents (because it is four P5's away from C). This would widen the gap between C and E from 400 cents (14 cents sharper than a just major third) to 480 cents (18 cents flatter than a just perfect fourth).

William Sethares' composition *C to Shining C* uses this tuning progression (<http://tinyurl.com/6ksc9b>).

### Temperament Modulations

A temperament modulation involves a change of temperament via a *pivot tuning*.

12-TET tuning falls within the tuning continua of both the syntonic and schismatic temperaments. In 12-TET tuning, the diminished fourth (d4) and major third (M3) have the same width: 400 cents. This makes the schismatic and syntonic temperaments enharmonically equivalent in 12-TET, so 12-TET can be used as a pivot tuning between the schismatic and syntonic temperaments.

One could start a tuning progression in 31-TET/syntonic, using syntonic note-choice rules. For example, one would play major triads as (root, M3, P5).

Then one could tune up to 12-TET, and indicate a change in temperament to schismatic through a user interface gesture. One would then also change note-choice to match the new temperament, playing (for example) major triads as (root, d4, P5). This change of temperament would not be detectable in 12-TET, because the syntonic & schismatic temperaments are enharmonic in that tuning.

Then, one could tune up to 53-TET, continuing to use schismatic note-choice rules, playing musical patterns that are indigenous to the schismatic temperament but not to

the syntonic, to emphasize the effect of the temperament modulation.

One could then either tune back to 31-TET/syntonic by using 12-TET as a pivot tuning, or modulate from 53-TET/schismatic to some other temperament via some other pivot tuning. Cyclical temperament modulations that pass through a number of different temperaments before returning to the initial temperament would enable *temperament progressions*. This is virgin territory for creative artists in the 21<sup>st</sup> Century.

The polyphonic tuning bend, tuning progression, and temperament modulation are just three examples of the new musical effects enabled by the Wicki note-layout's isomorphic exposure of the invariant properties of music. You can explore them using your computer keyboard and free applications available online.<sup>2</sup>

Changing tuning dynamically with consistent fingering is a novel and useful feature, but many of the resulting tunings are not consonant when played using harmonic timbres. To have the option of consonance in any tuning, tuning and timbre must be tempered together.

### Dynamic Timbre

In Dynamic Tonality, a given temperament defines (through its generators and comma sequence) an invariant pattern which is used to electronically temper both tuning and timbre dynamically in real time. Aligning tuning with timbre (or *vice versa*) maximizes consonance [17].

The syntonic temperament associates the Harmonic Series'

- 2<sup>nd</sup> partial with the octave (P8),
- 3<sup>rd</sup> partial with the tempered perfect fifth (P5),
- 5<sup>th</sup> partial with the tempered major third (M3), which is defined to be four P5's wide, minus two P8's (*i.e.*,  $[(4 * P5) + (-2 * P8)]$ ).

Note that the width of the M3 is defined by a combination of P5's and P8's (those being the temperament's generators). The same is true for every interval in the syntonic temperament.

In Dynamic Tonality, the structure of a tempered timbre is modified away from the Harmonic Series (*exactly* as a tempered tuning is modified away from Just Intonation) by adjusting its partials to maintain their alignment with the current tuning's notes.

---

<sup>2</sup> Demonstration implementation of a Dynamic Tonality synthesizer: <http://tinyurl.com/6325as>

For example, in 12-TET tuning,

- the P8 is untempered at 1200 cents wide, aligning with the timbre's 2<sup>nd</sup> partial 1200 cents above the 1<sup>st</sup> partial (*i.e.*, the fundamental);
- the P5 is tempered to be 700 cents wide, and the timbre's 3<sup>rd</sup> partial is placed 700 cents above its 2<sup>nd</sup> partial; and
- the M3 is tempered to be 400 cents wide (*i.e.*,  $[(4*700) + (-2*1200)] = [(2800) + (-2400)] = 400$ ), and the timbre's 5<sup>th</sup> partial is placed 400 cents above its 4<sup>th</sup> partial.

Similarly, in \_-comma meantone tuning,

- the P8 is untempered at 1200 cents in width, aligning with the timbre's 2<sup>nd</sup> partial 1200 cents above the 1<sup>st</sup> partial (*i.e.*, the fundamental);
- the P5 is tempered to be 696.6 cents wide, and the timbre's 3<sup>rd</sup> partial is placed 696.6 cents above its 2<sup>nd</sup> partial, and
- the M3 is tempered to be 386.4 cents wide (*i.e.*,  $[(4*696.6) + (-2 * 1200)] = [(2786.4) + (-2400)] = 386.4$ ), and the timbre's 5<sup>th</sup> partial is placed 386.4 cents above its 4<sup>th</sup> partial.

All along the syntonic tuning continuum, the width of the tempered P5 (combined with the unchanging width of the P8 at 1200 cents and the unchanging rules of the syntonic temperament, such as  $M3 = [(4*P5) + (-2*P8)]$ ), determines the placement of the timbre's partials and the width of the tuning's intervals. This paired tempering of tuning and timbre maximizes the consonance of tonal intervals all along the tuning continuum.

Dynamic Tonality, combined with the Wicki note-layout, appears to offer a general solution to the problem of temperament [8]—*i.e.*, a solution that delivers both consonance and modulatory freedom across all keys and tunings, enabling the performer to slide smoothly among tunings with consistent fingering and without loss of consonance. It is *general* in that it works for every octave, key, and syntonic tuning; however, it is *specific* in that it only works for isomorphic button-fields driving compatible electronic synthesizers (not traditional acoustic instruments).

#### *Timbre Effects*

Dynamic Tonality enables efficient timbre manipulations that are relevant to the structure of tonality, including *sonance* and *primeness* [14].

#### **Sonance**

Dynamic Tonality enables one to adjust a timbre's partials to change their *sonance*—*i.e.*, consonance or dissonance—on the fly, in real time.

#### **Primeness**

Consider a harmonic timbre's 2<sup>nd</sup>, 4<sup>th</sup>, 8<sup>th</sup>, 16<sup>th</sup>, ...2<sup>nth</sup> partials. Their prime factorization contains only the number “two,” so they can be said to embody *twoness*. Likewise, the 3<sup>rd</sup>, 9<sup>th</sup>, 27<sup>th</sup>, ...3<sup>nth</sup> partials are factored only by three, and so can be said to embody *threeness*; while the 5<sup>th</sup>, 25<sup>th</sup>, 125<sup>th</sup>, ...5<sup>nth</sup> partials embody *fiveness*, and so on. On the other hand, the 10<sup>th</sup> partial can be factored into both 2 and 5, so it embodies both twoness and fiveness, while the 20<sup>th</sup> partial, being factored as  $2*2*5$ , can be said to embody twice as much twoness as fiveness.

In Dynamic Tonality, one can manipulate a tempered timbre to enhance its twoness, threeness, fiveness, etc.—generally, its *primeness*—on the fly.

Turning twoness down will lead to an odd-partial-only timbre like that of closed-bore cylindrical instruments (*e.g.*, the clarinet). Turning up the twoness would gradually re-introduce the even-numbered partials, creating a sound like that of open-bore cylindrical instruments (*e.g.*, flute or shakuhachi) or conical bore instruments (*e.g.*, the saxophone, bassoon, or oboe). Adjusting the other primenesses would produce different but highly-distinctive changes in timbre.

Likewise, one could use real-time timbre changes to emphasize the bluesy quality of a piece, phrase, or even a single note by turning up the sevenness (given the role of the 7<sup>th</sup> partial in defining the width of the “blue intervals” of the blues scale [11]).

#### **Dynamic Tonality and Alternative Controllers**

All isomorphic note-layouts share the property of fingering invariance over a tuning continuum. However, other mathematical properties [13] of the Wicki note-layout make it particularly well-suited for Dynamic Tonality. Also, the Wicki note-layout can be conveniently mapped to a standard computer keyboard, making it universally available to electronic musicians.

Fingering invariance is necessary for Dynamic Tonality, but it is not sufficient. Controlling tuning requires an “extra” degree of freedom in addition to the usual expressive variables such as pitch bend and volume. If, in addition, one wishes to independently control the above-described timbre effects, then additional degrees of freedom are required.

#### **THE THUMMER**

To the best of our knowledge, the only controller that meets the real-time performance requirements of



Dynamic Tonality, as described above, is the Thummer™ [15].

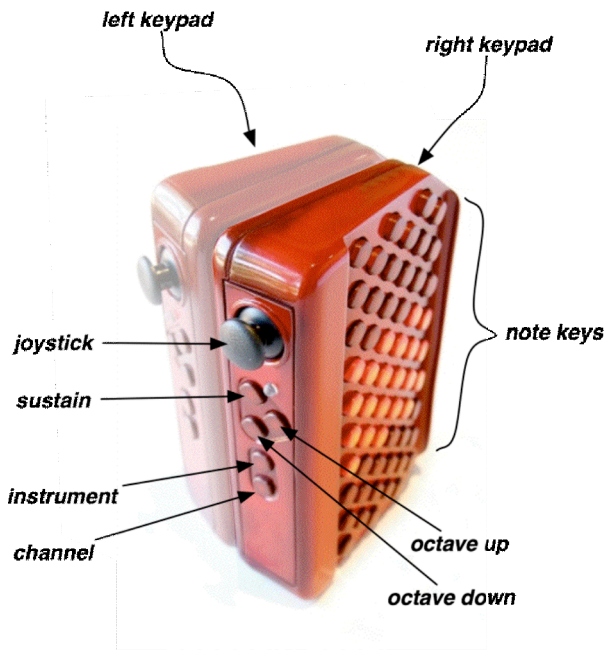


Figure 5: A prototype Thummer.

The Thummer places three octaves of 19 buttons per octave within the span of a single hand's fingers. Combining two such button-fields—one for each hand—produces a tiny controller, as shown in Figure 5 above. The Thummer is about the size of a thick paperback book, partially opened.

The Thummer's tiny size allows it to be supported by a brace affixed to one forearm, as shown in Figure 6 below, with the other arm remaining completely free.

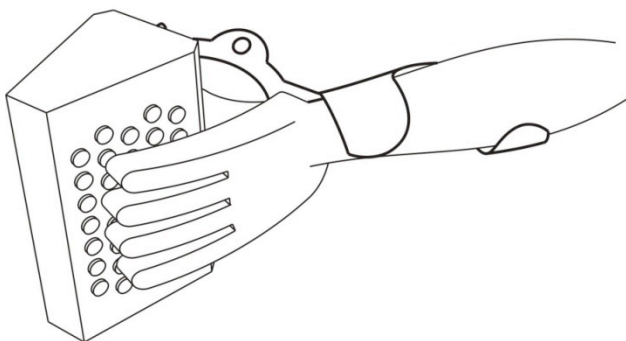


Figure 6: Sketch of prototype Thummer forearm brace.

The use of such a brace frees both hands' fingers to play their respective button-fields, and both hands' thumbs to control tiny joysticks like those on a video game controller. The instrument can also contain internal motion sensors, like those in Nintendo's Wii Remote and

Sony's SixAxis controllers (which sense acceleration and rotation around all three spatial axes).

The Thummer's two eponymous thumb-operated joysticks each have two axes, providing four degrees of freedom between them. These, plus six axes of motion sensing, provide intimate control of up to ten degrees of freedom. Thus, ten independent continuous control axes can be devoted to the control of any expressive parameter, including pitch bend, vibrato speed, vibrato depth, brightness, reverberation, etc. No other polyphonic musical instrument of which we are aware offers the ability to control this many degrees of freedom in real time while playing notes with the fingers of both hands.

### INTELLECTUAL PROPERTY

The Thummer is patent-pending in the USA and many international jurisdictions. However, the patent holder, Thumtronics, has been driven to the brink of bankruptcy by the 2008-2009 global financial crisis, so its patents are likely to fall into the public domain.

### CONCLUSIONS

Dynamic Tonality expands the time-honored framework of tonality to embrace previously inaccessible new creative frontiers. By combining the Wicki note-layout's isomorphic exposure of transpositional and tuning invariance, the electronic synthesizer's flexibility of tuning and timbre, and the Thummer's expressive power, Dynamic Tonality provides novel tonal resources that prepare art music for the 21<sup>st</sup> Century.

### REFERENCES

1. Barbour, M., 1951, *Tuning and Temperament: A Historical Survey*, ISBN: 978-0306704222.
2. Blackwood, E., *The Structure of Recognizable Diatonic Tunings*, 1986, Princeton Univ. Press, ISBN: 978-0691091297.
3. Braun, M., 2003, *Bell tuning in ancient China*, <http://web.telia.com/~u57011259/Zengbells.htm>
4. College Music Society, *2008 Call for Papers*, <http://www.music.org>
5. Erlich, P., *A middle path between just intonation and the equal temperaments, Part 1*, *Xenharmonikon* 18 (2006), 159-199.
6. Gaskins, R., 2003, *The Wicki System*, <http://www.concertina.com/gaskins/wicki>
7. Hassan Touma, H., 1996, *The Music of the Arabs*, trans. L. Schwartz. Portland, Oregon: Amadeus Press. ISBN 0-931340-88-8.
8. Isacoff, S., 2004, *Temperament: How Music Became a Battleground for the Great Minds of Western Civilization*, ISBN: 0-375-70330-6.

9. Jessup, L. 1983. *The Mandinka Balafon: An Introduction with Notation for Teaching*, ISBN: 978-0916421014.
10. Keislar, D., 1988, *History and Principles of Microtonal Keyboard Design*, Report No. STAN-M-45, Center for Computer Research in Music and Acoustics, Stanford University.
11. Kubik, G., 1999, *Africa and the Blues*, University Press of Mississippi, ISBN: 1-57806-146-6.
12. Milne, A., Sethares, W.A. and Plamondon, J., 2007, *Invariant Fingerings Across a Tuning Continuum*, Computer Music Journal, Winter 2007, Vol. 31, No. 4, Pages 15-32.
13. Milne, A., Sethares, W.A. and Plamondon, J., *Tuning Continua and Keyboard Layouts*, Journal of Mathematics and Music, Spring 2008, Vol. 2, No. 1, Pages 1-19.
14. Sethares, W. A., Milne, A., Tiedje, S., Prechtl, A., and Plamondon, J., *Spectral Tools for Dynamic Tonality and Audio Morphing*, Computer Music Journal, (accepted).
15. Paine, G., Stevenson, I., Pearce, A., *The Thummer Mapping Project (ThuMP)*, Proc. of the 7th International Conf. on New Interfaces for Musical Expression, 2007, Page 70-77.
16. Smith, G. W., 2006, *Comma Sequences*, <http://lumma.org/tuning/gws/commaseq.htm>
17. Sethares, W.A., 2005, *Tuning, Timbre, Spectrum, Scale*, ISBN: 1852337974.
18. Sundberg, J., A. Friberg, and L. Frydén. 1989. *Rules for Automated Performance of Ensemble Music*, *Contemporary Music Review* 3(1):89–109.