Joint Distributed Clustering and Ranging for Wireless Ad-Hoc Sensor Networks

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***Abstract*— This paper discusses a joint decentralized clustering and ranging algorithm for wireless ad-hoc sensor networks. Each sensor uses a random waiting timer and local criteria to determine whether to form a new cluster or to join a current cluster and utilizes the messages transmitted during hierarchical clustering to establish two-way communications so that clock calibration for distance estimation can be achieved. The algorithm operates without a centralized controller, it operates asynchronously, and does not require that the location of the sensors be known a priori. An analysis of the distance measurement, and the energy requirements of the algorithm are used to study the behaviors of the proposed algorithm. The performance of the algorithm is described analytically and via simulation.**

I. Introduction

Sensor location estimation is required in many sensor network applications [1]-[3]. Due to the low power, lower cost, and simple configuration requirements of wireless sensor networks, GPS devices, accurate synchronous clocks, and the installation of a base station may be precluded. However, when all sensors can measure the range to their neighbors, accurate relative location estimates are possible [4]-[6]. Moreover, wireless ad-hoc sensor networks are self-configuring distributed systems and, for reliability, should also operate without centralized control. In order to share information between sensors which cannot communicate directly, communication may occur via intermediaries in a multi-hop fashion. Scalability and the need to conserve energy lead to the idea of organizing the sensors hierarchically, which can be accomplished by gathering collections of sensors into clusters [7]-[10].

In previous work, the Clustering Algorithm via Waiting Timer (CAWT) [10] and The Distance Estimation via Asynchronous Clocks (DEVAC) [11] methods were applied separately to carry out the clustering and ranging tasks, respectively. This paper introduces a unified approach, the *Joint Distributed Clustering and Ranging Algorithm (JDCRA)*, which is applied at the network level to group sensors into clusters and to estimate the distances between pairs of sensors simultaneously. The JDCRA comprises the Modified CAWT algorithm using a local criterion to self-organize the network and the Modified DEVAC method (detailed in Section II) using bi-directional communications to bypass the need of synchronous clocking for accurate distance estimation based on time-of-arrival measurements. An estimation-theoretic analysis of the proposed measurement mechanisms is presented to assess the achievable ranging accuracy. The performance of the algorithm is investigated by simulation and numerical results are presented in a number of settings.

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II. The Joint Distributed Clustering and Ranging Algorithm (JDCRA)

A. Forming Clusters

1) The Modified CAWT: When the sensors of a network are first deployed, they may be partitioned into clusters using the modified CAWT from [10] with the waiting timer

$$WT_i^{(k+1)} = \beta \cdot WT_i^{(k)}, \quad (1)$$

where $WT_i^{(k)}$ is the waiting time of sensor *i* at time step k and $0 < \beta < 1$ is inversely proportional to the number of neighbors. Assume the initial value of the waiting time of sensor *i*, $WT_i^{(0)}$, is a sample from the distribution $C+\alpha\cdot U(0,1)$, where C and α are positive numbers, and $U(0,1)$ is a uniform distribution. If a sensor receives a $Hello$ message from a neighboring sensor before transmitting its *Hello*, wait r time units and the processing delay for clock calibration and then transmit a replied $Hello$ message; otherwise, wait r time units and transmit a $Hello$ message. As the random waiting timer expires and none of the neighboring sensors are in a cluster, then sensor *i* declares itself a clusterhead. It then broadcasts a message including the notification of its neighbors that they are assigned to join the new cluster with ID *i* and the information of the distance estimation.

2) Hierarchical Clustering: After applying the Modified CAWT, there are three different kinds of sensors: (1) the clusterheads (2) sensors with an assigned cluster ID (3) sensors which become 2-hop sensors. These sensors will join any nearby cluster after τ seconds where τ is a constant chosen to be larger than all of the waiting times. Thus, the topology of the ad-hoc network is now represented by a hierarchical collection of clusters. In the proposed JDCRA, each sensor initiates 2 rounds of local flooding to its 1-hop neighboring sensors, one for broadcasting sensor ID and the other for broadcasting cluster ID, to select clusterheads and form 2-hop clusters. Hence, the time complexity is $O(2)$ rounds. The complete procedure of the initialization phase is outlined in Table I.

B. Two-Way Ranging

The most straightforward method of estimating the distance between sensors directly measures the time required for a signal to propagate between the sensors. For low-powered sensors where the communication range is limited to a few hundred

TABLE I

THE JDCRA: AN ALGORITHM FOR CLUSTERING AND RANGING

1. Each sensor initializes a random waiting timer with a value $WT_i^{(0)}$.
2. A *Hello* message consists of:
   (1) the sensor ID of the sending sensor,
   (2) the cluster ID of the sending sensor,
   (3) information.
   (At the beginning, the cluster ID of each sensor is zero.)
3. Each sensor transmits the *Hello* message at random times:
   if a sensor receives a $Hello$ message from a neighboring sensor before transmitting its *Hello*
      (a)Draw a sample r from the distribution $\lambda \cdot WT_i^{(0)} \cdot U(0,1)$, where $0 < \lambda < 0.5$
      (b)wait r time units and the processing delay for clock calibration between pairs of sensors.
      (c)transmit the replied $Hello$ message.
   else
      wait r time units and then transmit the $Hello$ message.
4. Establish and update the neighbor identification:
   if a sensor receives a message of assigning a cluster ID at time step k
      (a) join the corresponding cluster.
      (b) draw a sample r' from the distribution $WT_i^{(k)} \cdot U(0,1)$.
      (c) wait r' time units and then send an updated $Hello$ message with the new cluster ID.
      (d) stop the waiting timer. (Stop!)
   else
      count the number of neighboring sensors.
   end
5. Decrease the random waiting time according to equation (1).
6. Clusterhead check:
   if $WT_i = 0$ and the neighboring sensors are not in another cluster
      (a) broadcast itself to be a clusterhead.
      (b) assign the neighboring sensors to cluster ID *i*. (Stop!)
   elseif $WT_i = 0$ and some of the neighboring sensors are in other clusters
      join any nearby cluster after τ seconds, where τ is greater than any possible waiting time. (Stop!)
   else
      go to Step 3.
   end

meters, the distance must be estimated to sub-meter accuracy. This can be accomplished using accurate clocks, but these may be more expensive than desired in the network application. The Distance Estimation via Asynchronous Clocks (DEVAC) method [11] helps to alleviate the need for highly accurate synchronous clocking.

1) The Modified DEVAC Method: Instead of transmitting a pulse signal twice to adjust the clock skew in the DEVAC method, sensor A transmits a ranging sequence using Ultra-Wideband (UWB) signaling to achieve clock calibration. Suppose that sensors A and B are equipped with clocks (oscillators) that are assumed to be asynchronous in both frequency and phase. Denote t_i^a and t_j^b as the time stamps in sensors A and B, respectively; let t_{del}^a and t_{del}^b be the delay time in sensors A and B, respectively; t_{ab} is the signal propagation time. The estimation of the Modified DEVAC method proceeds as shown in Figure 1:

a. Sensor A transmits a *Hello* message, which is a ranging sequence comprising K symbols and containing the times t_0^a and t_1^a (the times indicated on its clock at the start and the end of the transmission, respectively).
b. Sensor B receives the first symbol at time t_2^b (which is t_{ab} seconds after it is transmitted) and receives the last symbol at time t_3^b.
c. Sensor B calibrates its clock to A's using the differences $t_1^a - t_0^a$ (which is known from A's message) and $t_3^b - t_2^b$ (the arrival times).
d. Some time t_{del} later, sensor B transmits the time $t_{del}^a = z \cdot t_{del}^b$ that has elapsed since reception of A's message along with the time stamp t_4^b (the time on B's clock when it transmits). These times are adjusted (if necessary) using the scale factor $z = \frac{t_1^a - t_0^a}{t_3^b - t_2^b}$.
e. Sensor A receives the replied *hello* message from sensor B when its clock reads t_5^a (the time indicated on its clock at the start of the reception). The transmission time t_{ab} can be calculated as

$$t_{ab} = \frac{t_5^a - t_1^a - t_{del}^a}{2}.$$

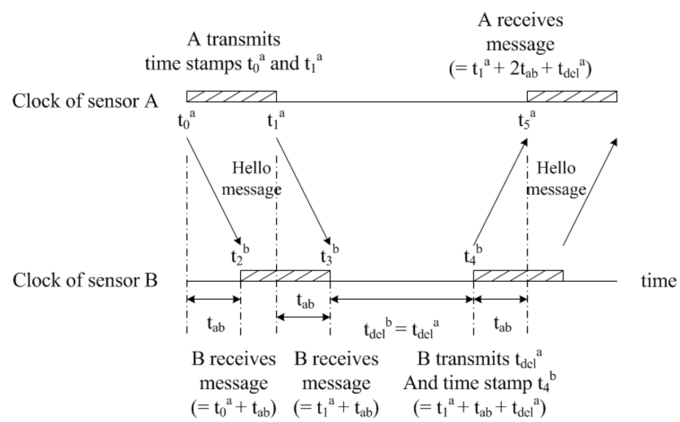

Fig. 1. The Modified DEVAC Method: Sensor A receives its reply at t_5^a. This is equal to $t_1^a + 2t_{ab} + t_{del}^a$, from which A can estimate t_{ab} and hence the distance. In this variation, sensor B can calculate the difference between its clock ($t_3^b - t_2^b$) and A's clock using the time stamped information in A's messages ($t_1^a - t_0^a$).

2) Analysis of the Distance Estimation: This paper assumes that only the line-of-sight (LOS) path exists. If multipath interference exists, then more complex signaling schemes can be used [20]. The accuracy of the distance measurement is analyzed as a function of the accuracy of the clock by deriving an approximate distribution for the estimation based on Figure 1. Quantitative expressions are provided to demonstrate the operation of the Modified DEVAC method. The random variable T represents the sensor estimate of the true *t*; thus T_{ab} is an estimate of the true time t_{ab} and T_i^a is the estimate of the time t_i^a as measured by the clock of sensor A. The estimated transmission time is

$$T_{ab} = \frac{T_5^a - T_{del}^a - T_1^a}{2}. \quad (2)$$

Since sensor B calibrates its clock to A's using time differences,

$$T_{del}^a = Z \cdot T_{del}^b, \quad (3)$$

where $T_{del}^b = T_4^b - T_3^b$ and

$$Z = \frac{T_1^a - T_0^a}{T_3^b - T_2^b} \quad (4)$$

is a scale factor that represents how much faster or slower clock A moves than clock B.

For the purpose of analysis, assume that all measurements T_i^a and T_j^b are independent normal random variables with the same variance σ^2 caused by the measurement error in the clock:

$$T_i^a \sim N(t_i^a, \sigma^2) \quad \text{for } i = 0, 1, 5. \tag{5}$$

$$T_j^b \sim N(t_j^b, \sigma^2) \quad \text{for } j = 2, 3, 4. \tag{6}$$

This normality assumption is justified in [12] when the clock skew is small.

Hence the random variable Z is the ratio of two normal random variables. As shown in [13] and [14], under reasonable conditions on the distributions, Z is well approximated by

$$Z \sim N(\mu_Z, \sigma_Z^2) \tag{7}$$

with $\mu_Z = \frac{\mu_1}{\mu_2}$ and $\sigma_Z^2 = \frac{2\sigma^2}{\mu_2^2}\left(1 + \left(\frac{\mu_1}{\mu_2}\right)^2\right)$, where $\mu_1 = t_1^a - t_0^a$ and $\mu_2 = t_3^b - t_2^b$. For this Gaussian approximation to hold, μ_2 must be biased away from zero and the ratio μ_2/σ^2 must be large. These are reasonable assumptions in the sensor communication application.

From (3) and (7), T_{del}^a can be viewed as the product of two normal random variables. Since the measurement errors are small, [15] shows that the distribution of T_{del}^a can be sensibly approximated by

$$T_{del}^a \sim N\left(\mu_Z t_{del}^b, 2\mu_Z^2\sigma^2 + {t_{del}^b}^2 \sigma_Z^2\right) \tag{8}$$

when μ_Z/σ_Z and $\mu_{T_{del}^b}/\sigma_{T_{del}^b}$ are large, which is a reasonable assumption in this case.

Using the above analysis and referring to (2), the distribution of T_{ab} is

$$T_{ab} \sim N\left(\mu_{T_{ab}}, \sigma_{T_{ab}}^2\right), \tag{9}$$

where $\mu_{T_{ab}} = \frac{1}{2}(t_5^a - \mu_Z t_{del}^b - t_1^a)$ and $\sigma_{T_{ab}}^2 = \frac{1}{4}\left[(2 + 2\mu_Z^2)\sigma^2 + {t_{del}^b}^2 \sigma_Z^2\right]$. Note that the mean of random variable T_{ab} is the true value of the transmission time between sensors A and B and the variance of T_{ab} depends on the variance of the timing measurement σ^2, the characteristic of the clock-adjustment factor (4), and the time delay t_{del}^b.

Finally, the distribution of the distance measurement D_{ab} is given by

$$D_{ab} \sim N\left(c\mu_{T_{ab}}, c^2\sigma_{T_{ab}}^2\right) \tag{10}$$

since the transmission distance is the product of the transmission speed c and the transmission time. Observe that the mean of random variable D_{ab} is the true value of the distance, showing that the estimator is unbiased. Numerical results are presented in Section IV.

Results from [16]-[17] relate the accuracy of synchronous distance estimates to the signal-to-noise ratio (SNR) and the effective bandwidth of the signal. The expression in (10) is the added inaccuracy due to the asynchronous clocking mechanism. For the ranging method, the fundamental limitation on the accuracy of the estimates is related to the form of the signal and the clock, including the signal bandwidth, the SNR, and the timing calibration. Assume that the random range error and range bias error from propagation conditions are negligible. The range-measurement accuracy may be characterized by the measurement error, σ_R, given by the root-sum-square of the error components.

$$\sigma_R = \left(\sigma_S^2 + \sigma_{clock}^2\right)^{1/2}, \tag{11}$$

where σ_S is the SNR-dependent random range measurement error,

$$\sigma_S \geq \frac{c}{2\beta_e\sqrt{2\text{SNR}}}, \tag{12}$$

where β_e is the effective bandwidth of the signal [17], and σ_{clock} is the clock-dependent random range measurement error, $c\sigma_{T_{ab}}$.

III. Analysis of Energy Consumption

This section considers the energy consumption of the JD-CRA method assuming homogenous sensors. The total power requirements include the power required to transmit messages E_T, the power required to receive messages E_R, and the power required to process messages E_{pro}.

In the initialization phase, each sensor broadcasts a $Hello$ message to its neighboring sensors. Therefore, the number of transmissions N_{T_x} is equal to the number of sensors in the network, n, and the number of receptions N_{R_x} is the sum of the neighboring sensors of each sensor. That is,

$$N_{T_x} = n \text{ and } N_{R_x} = \sum_{j=1}^{n} N_j, \tag{13}$$

where N_j is the number of the neighboring sensors of sensor j.

As a sensor, say sensor i, meets the conditions of being a clusterhead, it broadcasts and assigns cluster ID i to its neighboring sensors. Its neighboring sensors then transmit a signal to their neighbors to update cluster ID information. During this clustering phase, $(1 + N_i)$ transmissions and $(N_i + \sum_{j\in C_i} N_j)$ receptions are executed, where C_i is the index set of neighboring sensors of sensor i. This procedure is applied to all clusterheads and their cluster members. Now let $N_{T_x}^c$ and $N_{R_x}^c$ denote the number of transmissions and receptions for all clusters, respectively. Hence,

$$N_{T_x}^c = \sum_{i\in I}(1 + N_i), \tag{14}$$

$$N_{R_x}^c = \sum_{i\in I}(\sum_{j\in C_i} N_j + N_i), \tag{15}$$

where I is a index set of clusterheads. Therefore, the total number of transmissions N_T, the total number of receptions N_R, and the total number of information processing for clock calibration N_{pro} are

$$N_T = N_{T_x} + N_{T_x}^c = n + \sum_{i\in I}(1 + N_i), \tag{16}$$

$$N_R = N_{R_x} + N_{R_x}^c = \sum_{j=1}^{n} N_j + \sum_{i\in I}(\sum_{j\in C_i} N_j + N_i). \tag{17}$$

$$N_{pro} = \sum_{i\in I}(\sum_{j\in C_i} N_j + N_i). \tag{18}$$

From (16), (17), and (18), the total energy consumption, E_{total}, for cluster formation and distance estimation in the wireless sensor network is

$$E_{total} = N_T \cdot E_T + N_R \cdot E_R + N_{pro} \cdot E_{pro}, \quad (19)$$

where E_{pro} is consumed by the clock calibration or propagation time calculation.

Observe that the above analysis is suitable for any transmitting range. However, overly small transmission ranges may result in isolated clusters whereas overly large transmission ranges may result in a single cluster. Therefore, in order to optimize energy consumption and encourage linking between clusters, it is sensible to consider the minimum transmission power (or range R) which will result in a fully connected network [18]. The performance of the total energy consumption of the JDCRA with different selections of R is examined via simulation.

IV. Simulations and Numerical Results

A. Ranging using UWB Communications

This subsection demonstrates the performance of the various distance measurement methods. Assume that the propagation time is $t_{ab} = 10^{-7}$s (i.e. the true distance is $d_{ab} = 30$ m) for all distance measurement settings. Note that these settings may represent a reasonable transmission range for many wireless sensor applications as in the emerging ZigBee standard.

The first set of numerical results evaluates the critical timing parameters t_i^a and t_j^b in the modified DEVAC method to determine the required level of timing resolution (i.e. the standard deviation of the time measurement σ). Figure 2 (left) shows the typical performances of time and distance measurement using (10) with the parameters detailed in the caption and the clocks providing a resolution of 1 ns and 10 ns, respectively.

The second set of numerical results shows the distribution of the best possible ranging accuracy using different UWB signal formats with synchronous clocking. Based on the characteristics of UWB signaling [19] and assuming that channel transfer function is independent of frequency and dependent upon distance between transmitter and receiver as the inverse of the square of distance d_{ab}, equation (12) can be further expressed by

$$\sigma_s \geq \frac{c \cdot d_{ab}}{4\pi} \sqrt{\frac{3N_0}{T_{obs} G_0 (f_H^3 - f_L^3)}}, \quad (20)$$

where T_{obs} is the observation time ($T_{obs} = t_1^a - t_0^a$), $N_0 = 0.5 \times 10^{-14}$ W/Hz, $G_0 = 7.413 \times 10^{-14}$ W/Hz regulated by FCC [19], and f_H and f_L are the highest and lowest frequency of UWB frequency bands, respectively. Note that the ranging accuracy in different UWB signal formats are related to the difference in bandwidth and the center frequency. Figure 2 (right) depicts that the DS-UWB high band signal format has the best ranging performance with time synchronization.

The third set of numerical results demonstrates the ranging accuracy related to SNR, signal bandwidth, and clock calibration using UWB communications. Figure 3 illustrates the best possible ranging performance using the JDCRA approach with UWB communications. It suggests that based on the same observation time the DS-UWB signaling has better ranging accuracy than the MB-OFDM signaling.

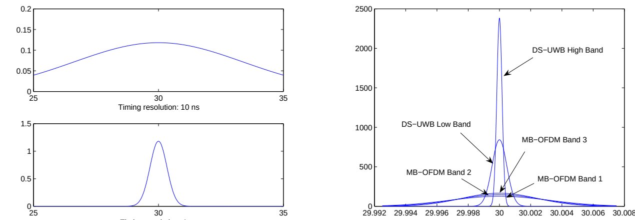

Fig. 2. The distribution of distance measurement using the JDCRA Method with a timing resolution of 10 ns (left top) and 1 ns (left bottom): $t_{ab} = 10^{-7}, t_4^b = 0.3 + 2.92\ \mu$s$, t_3^b = 0.3 + 1.92\ \mu$s$, t_2^b = 0.3, t_1^a = 0.25 + 1.83\ \mu$s, and $t_0^a = 0.25$ (left); the distribution of the ranging accuracy using different UWB signal formats with synchronous clocking (right).

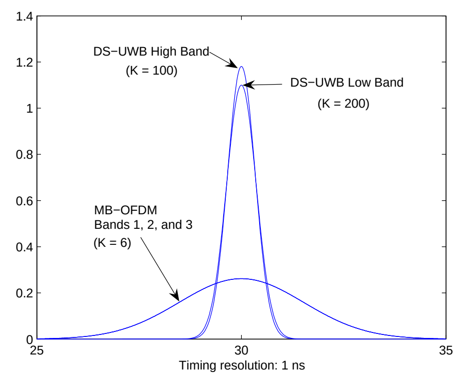

Fig. 3. The performance of the ranging accuracy using different UWB signal formats with the same observation time $T_{obs} = 1.83\ \mu$s and the parameters detailed in the caption of Figure 2.

B. Cluster Formation

The fourth set of experiments examines the variation of the average number of clusterheads with respect to the ratio *R/l*. With random waiting time parameters $C = 100, \alpha = 10$, and $\beta = 0.9$, Figure 4 depicts a typical run of the algorithm in a random network of 100 sensors with $R/l = 0.175$. The result shows that each cluster is a collection of sensors which are up to 2 hops away from a clusterhead.

Figure 5 shows the relationship between the average number of clusterheads and the *R/l* ratio with varying the number of sensors. The average number of clusterheads in each case is the sample mean of the results of 200 typical runs. Observe that the average number of clusterheads decreases as the ratio *R/l* increases (i.e. the transmission power increases). Since larger transmission power allows larger radio coverage, a clusterhead has more cluster members, which reduces the number of clusters in the network.

The last set of experiments considers the total energy consumption of the JDCRA. Assume that the communication channel is error-free. Since each sensor does not need to retransmit any data, two transmissions are executed, one

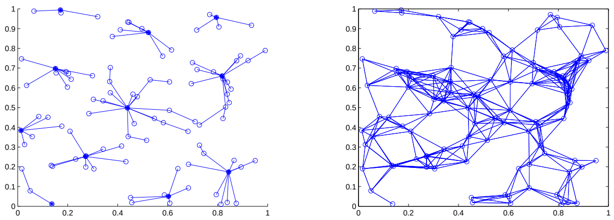

Fig. 4. Clusters are formed in a random network of 100 sensors with $R/l = 0.175$ (left); the connectivity of the network (right).

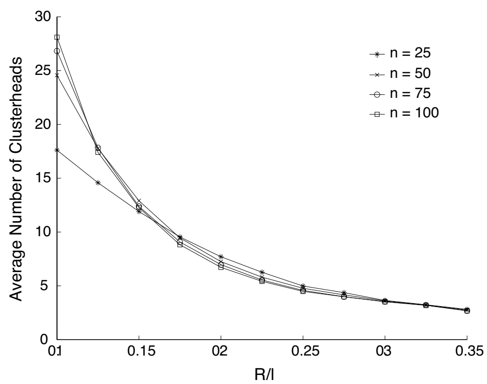

Fig. 5. Average number of clusterheads as a function of the ratio R/l.

for broadcasting the existence and the other for assigning a cluster ID to its cluster members or updating the cluster ID information of its neighbors. Hence, the total number of transmissions is $2n$. Under these circumstances, sensor *i* will receive $2N_i$ messages. Then, the total number of receptions is $2\sum_{i=1}^{n} N_i$. Figure 6 shows the average receptions of random networks after applying the proposed algorithm.

V. Conclusions

This paper proposes a novel approach for the simultaneous clustering and ranging of sensors and provides a description of an algorithm suitable for the infrastructure requirements of wireless sensor networks. The algorithm has several advantages over standard approaches since it requires no GPS and no synchronous clocks. For the hierarchical cluster-based network structure, local time synchronization is achieved by referencing to the clock of a clusterhead while global calibration can be achieved by (relatively sparse) communication between clusterheads.

The analytical portions of the paper presume that only the LOS path exists and future plans involve generalizing the method to allow for multipath channels, to consider certain failure scenarios, and to explore the influence of time synchronization problem on the network operation.

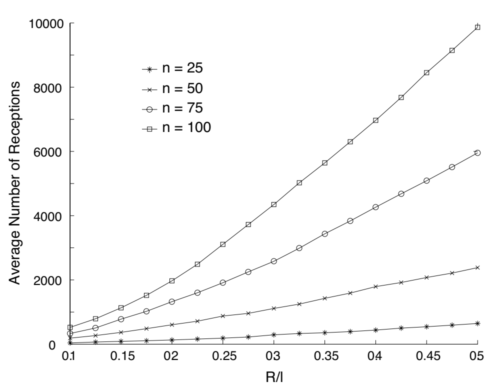

Fig. 6. The number of receptions in random networks as a function of the number of sensors and R/l ratio.

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