

9.21 a) From Table 9.2, row 6,

$$X(s) = \frac{1}{s+2} + \frac{1}{s+3} = \frac{2s+5}{s^2+5s+6}$$

ROC: $\text{Re } s > -2$ and $\text{Re } s > -3$

$$\Leftrightarrow \text{Re } s > -2$$

b) Table 9.2, rows 6, 14:

$$X(s) = \frac{1}{s+4} + \frac{5}{(s+5)^2+25} = \frac{s^2+15s+70}{s^3+14s^2+90s+200}$$

ROC: $\text{Re } s > -4$, $\text{Re } s > -5$

$$\Leftrightarrow \text{Re } s > -4$$

c) Table 9.2, row 7:

$$X(s) = -\frac{1}{s-2} - \frac{1}{s-3} = -\frac{2s-5}{s^2-5s+6}$$

ROC: $\text{Re } s < 2$, $\text{Re } s < 3$

$$\Leftrightarrow \text{Re } s < 2$$

$$d) x(t) = f e^{-2|t|} = f e^{-2t} u(t) + f e^{2t} u(-t)$$

Table 9.2, rows 8, 9 ($n=2$):

$$X(s) = \frac{1}{(s+2)^2} - \frac{1}{(s-2)^2} = -\frac{8s}{s^2 - 8s + 16}$$

$$\text{ROC: } -2 < \text{Re } s < 2$$

e) Similar to d),

$$x(t) = f e^{-2t} u(t) - f e^{2t} u(-t)$$

$$X(s) = \frac{1}{(s+2)^2} + \frac{1}{(s-2)^2} = \frac{2(s^2 + 4)}{s^2 - 8s + 16}$$

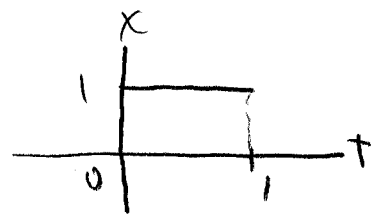
$$\text{ROC: } -2 < \text{Re } s < 2$$

$$f) x(t) = -f e^{2t} u(-t)$$

$$X(s) = \frac{1}{(s-2)^2}$$

$$\text{ROC: } \text{Re } s < 2$$

$$g) \quad x(t) = u(t) - u(t-1)$$



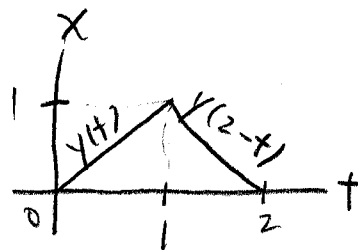
From the time-shift property,

$$X(s) = \frac{1}{s} - \frac{e^{-s}}{s} = \frac{1 - e^{-s}}{s}$$

$$\text{ROC: } -\infty < \text{Re } s < \infty$$

h) Let $y(t) = t(u(t) - u(t-1))$. Then

$$x(t) = y(t) + y(2-t).$$



Applying the frequency-differentiation, time-shift, and time-scaling properties to g), we obtain

$$Y(s) = -\frac{d}{ds} \left(\frac{1 - e^{-s}}{s} \right) = \frac{1 - (s+1)e^{-s}}{s^2}$$

$$X(s) = Y(s) + e^{-2s} Y(-s)$$

$$= \frac{1 - (s+1)e^{-s}}{s^2} + e^{-2s} \frac{1 + (s-1)e^s}{s^2}$$

$$= \left(\frac{1 - e^{-s}}{s} \right)^2, \quad -\infty < \text{Re } s < \infty$$

i) Table 9.2, rows 1, 2:

$$X(s) = 1 + \frac{1}{s} = \frac{s+1}{s}$$

$$\operatorname{Re} s > 0$$

j) From time-scaling and i),

$$X(s) = \frac{1}{3} \left(1 + \frac{1}{s/3} \right) = \frac{1}{3} \frac{s+3}{s}$$

$$\operatorname{Re} s > 0$$