

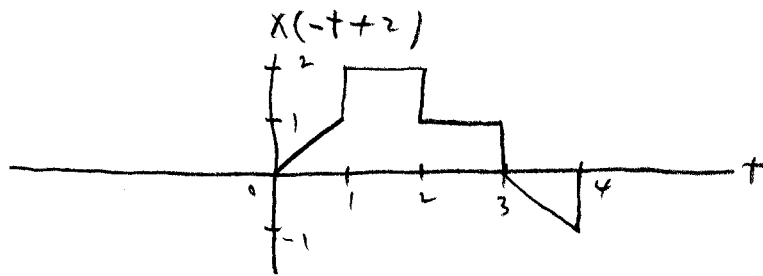
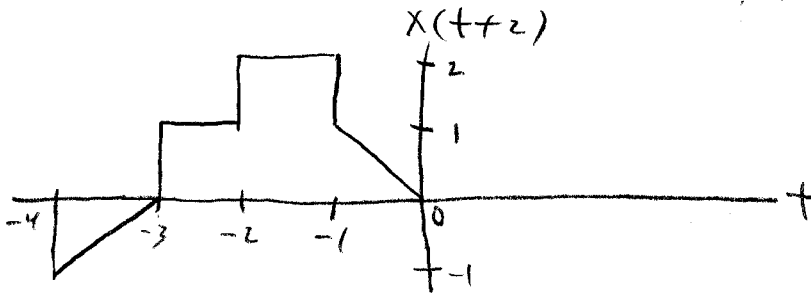
$$\begin{aligned}
1.3 \text{ c)} \quad P &= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt \\
&= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \cos^2 t \, dt \\
&= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \frac{1 + \cos 2t}{2} \, dt \\
&= \lim_{T \rightarrow \infty} \frac{1}{2T} \left( \frac{t}{2} + \frac{\sin 2t}{4} \right) \Big|_{-T}^T \\
&= \lim_{T \rightarrow \infty} \frac{1}{2T} \left( T + \frac{\sin 2T}{2} \right) \\
&= \lim_{T \rightarrow \infty} \left( \frac{1}{2} + \frac{\sin 2T}{2T} \right) \\
&= \frac{1}{2}
\end{aligned}$$

$x$  is a power signal, so  $E = 0$ .

$$\begin{aligned}
e) \quad P &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{k=-N}^N |x[k]|^2 \\
&= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{k=-N}^N |e^{j(\frac{\pi}{2}k + \frac{\pi}{8})}|^2 \\
&= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{k=-N}^N 1 \\
&= \lim_{N \rightarrow \infty} \frac{2N+1}{2N+1} \\
&= 1 \quad \Rightarrow \quad E = 0
\end{aligned}$$

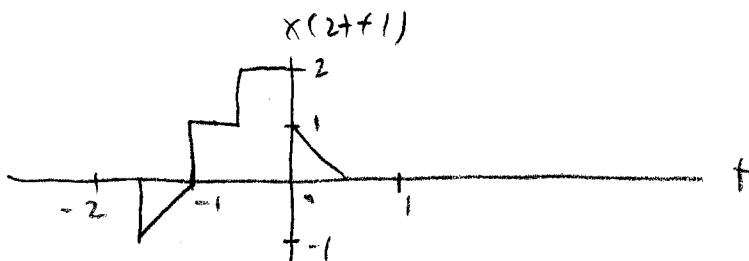
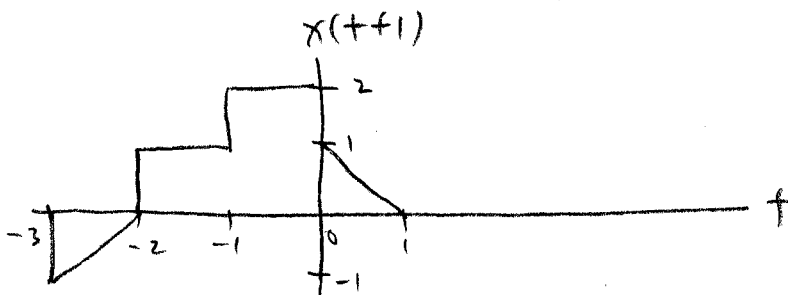
$$1.21 \text{ b) } x(2-t) = x(-t+2)$$

$\uparrow$  reflection       $\uparrow$  left shift  
 reflection      left shift



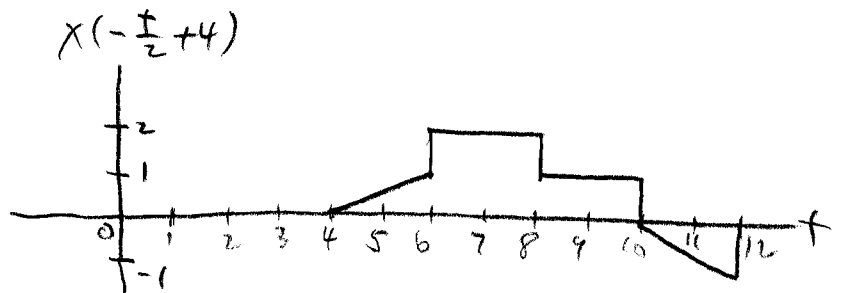
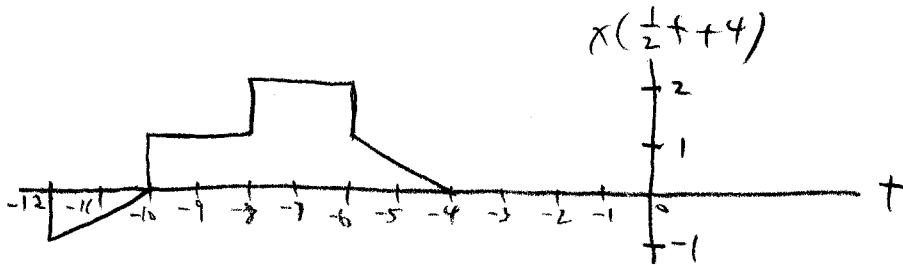
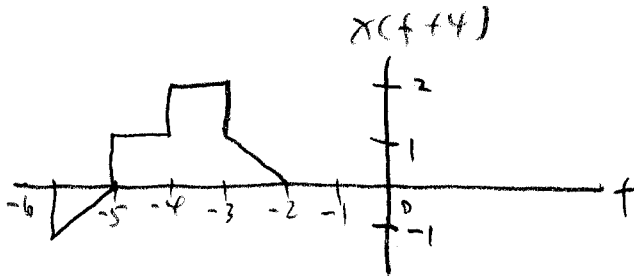
$$c) \quad x(2t+1) = x((2t)+1)$$

$\uparrow$  scaling       $\uparrow$  left shift  
 scaling      left shift

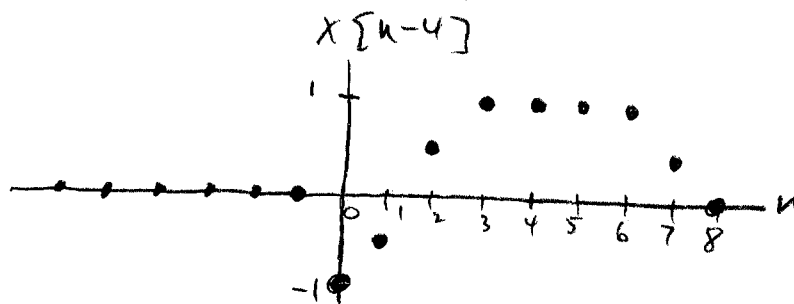


$$d) \quad x\left(4 - \frac{t}{2}\right) = x\left(\left(\frac{1}{2}(-t)\right) + 4\right)$$

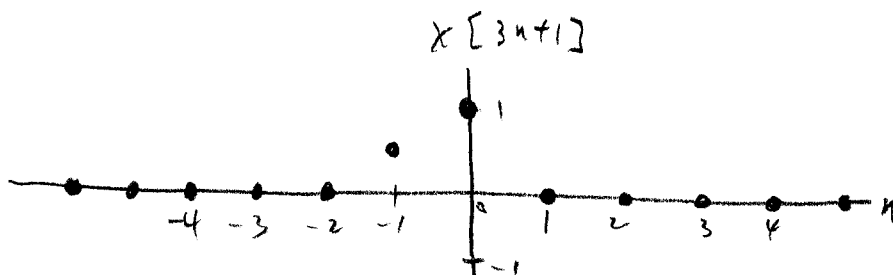
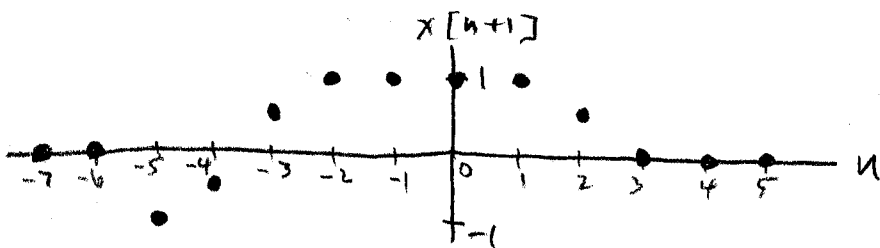
$\uparrow$     $\uparrow$     $\uparrow$   
 Scaling   reflection   left  
 shift



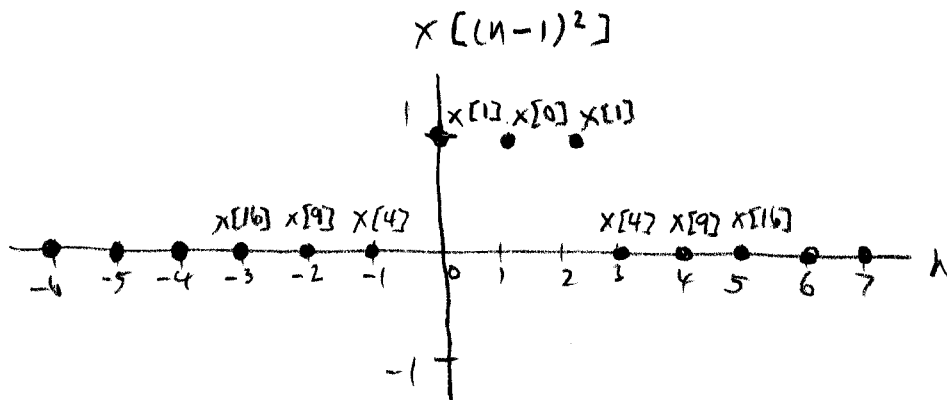
1-22 a)  $x[n-4]$   
 ↑  
 right  
 shift



d)  $x[3n+1] = x[(3n)+1]$   
 ↙ scaling ↘  
 ↖ left shift ↗



h)  $(n-1)^2$  cannot be obtained from the transformations discussed in class. The easiest way to solve this problem is to simply construct the graph point-by-point.



$$1.51 c) \quad \cos^2 \theta = \left( \frac{1}{2} (e^{j\theta} + e^{-j\theta}) \right)^2$$

$$= \frac{1}{4} (e^{j2\theta} + 2 + e^{-j2\theta})$$

$$\frac{1}{2} (1 + \cos 2\theta) = \frac{1}{2} \left( 1 + \frac{1}{2} (e^{j2\theta} + e^{-j2\theta}) \right)$$

$$= \frac{1}{4} (2 + e^{j2\theta} + e^{-j2\theta})$$

$$e) \quad \sin(\theta + \phi) = \frac{1}{2j} (e^{j(\theta+\phi)} - e^{-j(\theta+\phi)})$$

$$\sin \theta \cos \phi + \cos \theta \sin \phi = \frac{1}{2j} (e^{j\theta} - e^{-j\theta}) \cdot \frac{1}{2} (e^{j\phi} + e^{-j\phi})$$

$$+ \frac{1}{2} (e^{j\theta} + e^{-j\theta}) \cdot \frac{1}{2j} (e^{j\phi} - e^{-j\phi})$$

$$= \frac{1}{4j} \left( e^{j(\theta+\phi)} + e^{j(\theta-\phi)} - e^{j(\phi-\theta)} - e^{-j(\theta+\phi)} \right.$$

$$\left. + e^{j(\theta+\phi)} - e^{j(\theta-\phi)} + e^{j(\phi-\theta)} - e^{-j(\theta+\phi)} \right)$$

$$= \frac{1}{4j} (2e^{j(\theta+\phi)} - 2e^{-j(\theta+\phi)})$$

$$= \frac{1}{2j} (e^{j(\theta+\phi)} - e^{-j(\theta+\phi)})$$

$$1.53 \text{ b) Let } z_1 = x_1 + jy_1, z_2 = x_2 + jy_2.$$

$$\begin{aligned} z_1 z_2^* + z_1^* z_2 &= (x_1 + jy_1)(x_2 - jy_2) + (x_1 - jy_1)(x_2 + jy_2) \\ &= x_1 x_2 + y_1 y_2 + j(x_2 y_1 - x_1 y_2) \\ &\quad + x_1 x_2 + y_1 y_2 + j(x_1 y_2 - x_2 y_1) \\ &= 2(x_1 x_2 + y_1 y_2) \end{aligned}$$

$$\begin{aligned} 2 \operatorname{Re}(z_1 z_2^*) &= 2 \operatorname{Re}(x_1 + jy_1)(x_2 - jy_2) \\ &= 2(x_1 x_2 + y_1 y_2) \end{aligned}$$

$$\begin{aligned} 2 \operatorname{Re}(z_1^* z_2) &= 2 \operatorname{Re}(x_1 - jy_1)(x_2 + jy_2) \\ &= 2(x_1 x_2 + y_1 y_2) \end{aligned}$$

$$\begin{aligned} \text{f) } |z_1 z_2^* + z_1^* z_2| &\leq |z_1 z_2^*| + |z_1^* z_2| && \text{(triangle inequality)} \\ &= |z_1| |z_2^*| + |z_1^*| |z_2| \\ &= |z_1| |z_2| + |z_1| |z_2| \\ &= 2 |z_1| |z_2| \\ &= 2 |z_1 z_2| \end{aligned}$$

$$\begin{aligned}
 1.55 a) \quad \sum_{n=0}^9 e^{j\frac{\pi n}{2}} &= \sum_{n=0}^9 \left( e^{j\frac{\pi}{2}} \right)^n = \sum_{n=0}^9 j^n \\
 &= \frac{1-j^{10}}{1-j} = \frac{1-(-1)}{1-j} \\
 &= \frac{2}{1-j} = \frac{2(1+j)}{(1-j)(1+j)} \\
 &= \frac{2(1+j)}{2} = 1+j
 \end{aligned}$$

$$\begin{aligned}
 f) \quad \sum_{n=0}^{\infty} \left( \frac{1}{2} \right)^n \cos\left(\frac{\pi}{2}n\right) &= \sum_{n=0}^{\infty} \left( \frac{1}{2} \right)^n \left( \frac{1}{2} (e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}) \right) \\
 &= \frac{1}{2} \left( \sum_{n=0}^{\infty} \left( \frac{1}{2} e^{j\frac{\pi}{2}} \right)^n + \sum_{n=0}^{\infty} \left( \frac{1}{2} e^{-j\frac{\pi}{2}} \right)^n \right) \\
 &= \frac{1}{2} \left( \sum_{n=0}^{\infty} \left( \frac{j}{2} \right)^n + \sum_{n=0}^{\infty} \left( -\frac{j}{2} \right)^n \right) \\
 &= \frac{1}{2} \left( \frac{1}{1-\frac{j}{2}} + \frac{1}{1-(-\frac{j}{2})} \right) \\
 &= \frac{1}{2} \cdot \frac{2}{1+\frac{1}{4}} \\
 &= \frac{4}{5}
 \end{aligned}$$