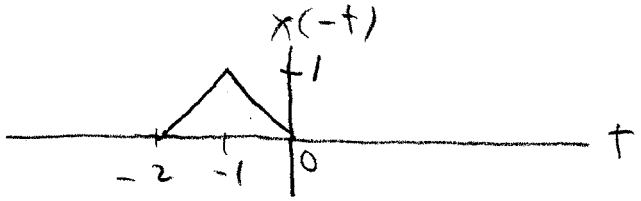
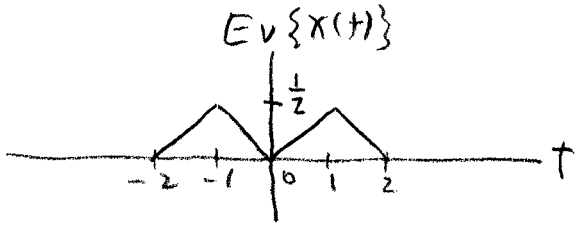
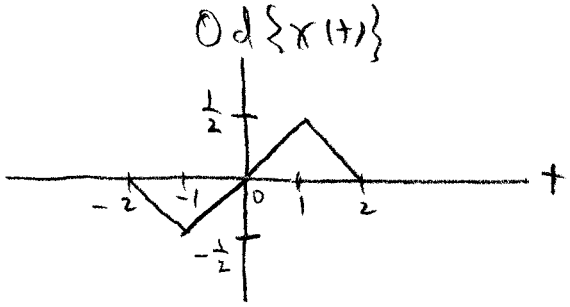
1.23 a) $Ev\{x(t)\} = \frac{1}{2}(x(t) + x(-t))$

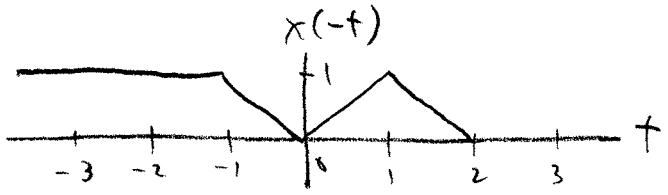


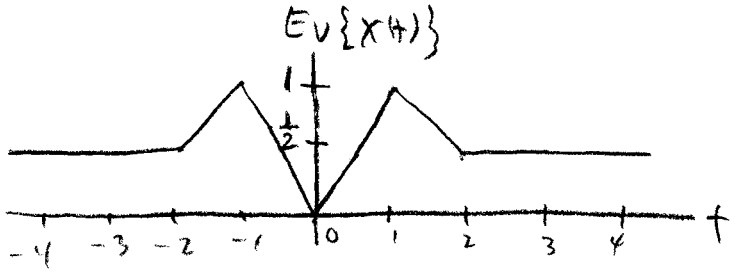


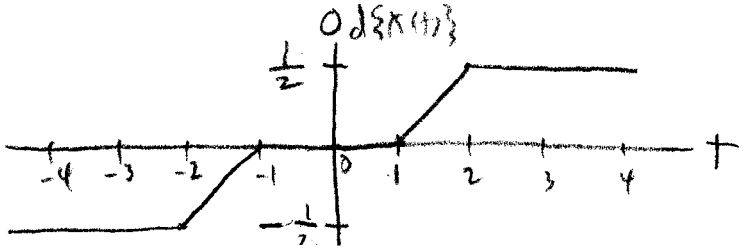
$Od\{x(t)\} = \frac{1}{2}(x(t) - x(-t))$



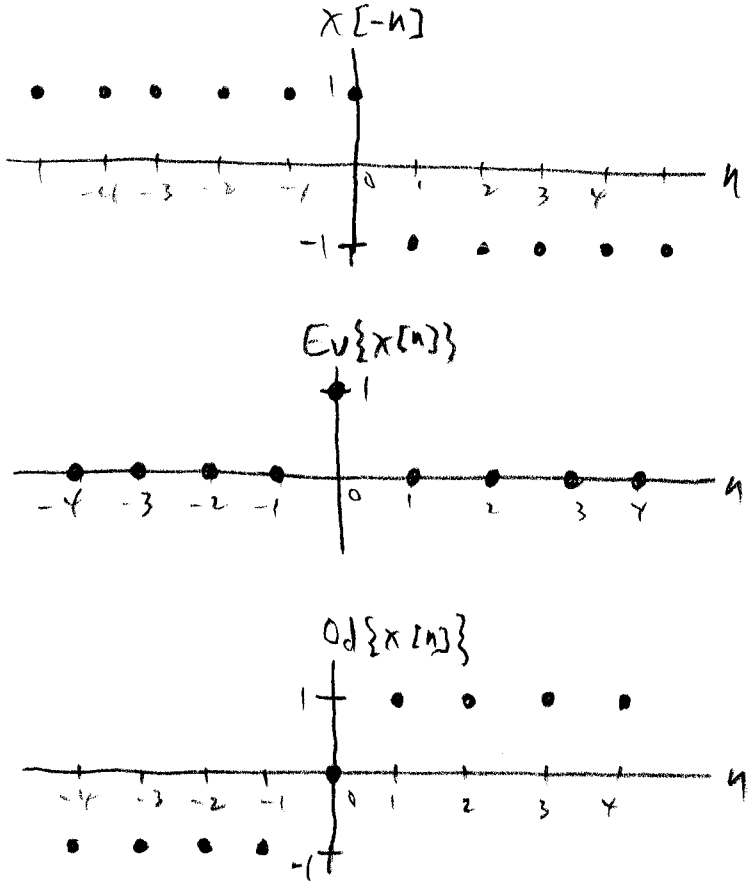
b)







1.24 a)



1.25 a) Time shifting does not change the fundamental period.

$$x(t) = 3\cos(4(t + \tfrac{\pi}{12}))$$

$$T_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

c) $$x(t) = \cos^2(2(t - \tfrac{\pi}{6})) = \frac{1 + \cos(4(t - \tfrac{\pi}{6}))}{2}$$

Time shifting, adding 1, and multiplying by $\frac{1}{2}$ does not change the fundamental period

$$T_0 = \frac{\pi}{2}$$

1.26 a) $\omega = \frac{6\pi}{7}$

$\frac{\omega}{2\pi} = \frac{3}{7}$ (rational)

$N_0 = 7$

b) $\omega = \frac{1}{8}$

$\frac{\omega}{2\pi} = \frac{1}{16\pi}$ (irrational)

$x[n]$ is not periodic.

c) If N is a period of $x[n]$, for every n x must satisfy

$$x[n+N] = x[n].$$

Set $n = 0$.

$$x[N] = x[0]$$

$$\cos\left(\frac{\pi}{8}N^2\right) = 1$$

$$\frac{\pi}{8}N^2 = 2\pi k \quad (k \text{ an integer})$$

$$\frac{N^2}{16} = k$$

If $\frac{N^2}{16}$ is an integer, N must be a multiple of 4.

$N = 4$ does not work, however, since

$$x[1] = \cos\frac{\pi}{8}$$

$$x[5] = \cos\frac{25\pi}{8}$$

$$= \cos\left(\frac{\pi}{8} + 3\pi\right)$$

$$= -\cos\frac{\pi}{8}$$

$$\neq \cos\frac{\pi}{8}$$

Try $N = 8$.

$$x[n+8] = \cos\left(\frac{\pi}{8}(n+8)^2\right)$$

$$= \cos\left(\frac{\pi}{8}n^2 + (n+4)(2\pi)\right)$$

$$= \cos\left(\frac{\pi}{8}n^2\right)$$

$$= x[n]$$

$$N_0 = 8$$

Plotting pairs gives the graph