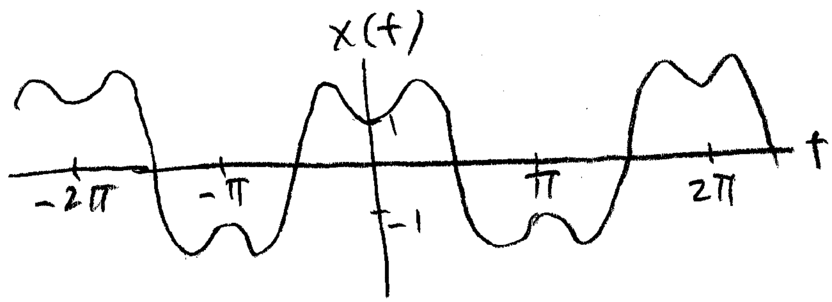
ECE 330
Fall 2009
Exam #1

Name: Solutions

Important Note: To receive full credit, you must show all your work and fully justify your answers.

1) For each of the following signals, determine whether $x(t)$ is periodic and find the fundamental period

a) $x(t) = 2\cos t - \cos 3t$



$T_0 = 2\pi$

From the definition,

$$x(t+2\pi) = 2\cos(t+2\pi) - \cos 3(t+2\pi)$$
$$= 2\cos t - \cos 3t$$
$$= x(t)$$

Periodic

b) $x[n] = e^{j\frac{5}{9}\pi n}$

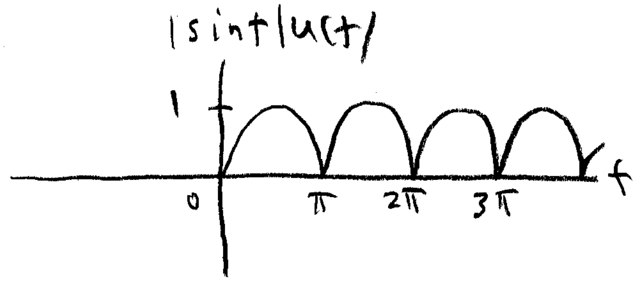
x is periodic iff $\frac{\Omega}{2\pi}$ is rational.

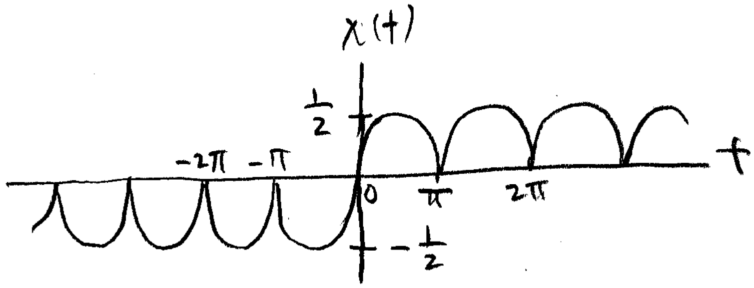
$$\frac{\Omega}{2\pi} = \frac{\frac{5}{9}\pi}{2\pi} = \frac{5}{18} = \frac{M_0}{N_0}$$

$N_0 = 18$

Periodic

c) $x(t) = \mathrm{Od}\{|\sin t|\, u(t)\}$





For any T and most t,

$x(t+T) \neq x(t)$.

Not Periodic

2) Consider the system given by

$$y(t) = \int_{-\infty}^{t} e^{-|\tau|} x(-\tau)\, d\tau.$$

Determine whether the system is

a) linear

$$\alpha x(t) \longrightarrow \int_{-\infty}^{t} e^{-|\tau|} (\alpha x(-\tau))\, d\tau = \alpha \int_{-\infty}^{t} e^{-|\tau|} x(-\tau)\, d\tau = \alpha y(t),$$

$$x_1(t) + x_2(t) \longrightarrow \int_{-\infty}^{t} e^{-|\tau|} (x_1(-\tau) + x_2(-\tau))\, d\tau$$

$$= \int_{-\infty}^{t} e^{-|\tau|} x_1(-\tau)\, d\tau + \int_{-\infty}^{t} e^{-|\tau|} x_2(-\tau)\, d\tau$$

$$= y_1(t) + y_2(t)$$

Yes

b) time-invariant

$$x(t+t_0) \longrightarrow \int_{-\infty}^{t} e^{-|\tau|} x(-\tau+t_0)\, d\tau$$

$$y(t+t_0) = \int_{-\infty}^{t+t_0} e^{-|\tau|} x(-\tau)\, d\tau$$

One way to show that $x(t+t_0) \not\to y(t+t_0)$ is to set $x(t) = 1$. Then

$$x(t+t_0) \longrightarrow \int_{-\infty}^{t} e^{-|\tau|}\, d\tau \neq \int_{-\infty}^{t+t_0} e^{-|\tau|}\, d\tau = y(t+t_0)$$

No

c) causal

Let $x(t) = u(t)$. For $t<0$,

$$y(t) = \int_{-\infty}^{t} e^{-|\tau|} u(-\tau) d\tau = \int_{-\infty}^{t} e^{\tau} d\tau = e^{t}$$

But the system is linear, and $x(t) = 0$ for $t<0$, so causality requires $y(t) = 0$ for $t<0$.

No

d) dynamic (has memory)

noncausal $\Rightarrow$ dynamic

Yes

e) BIBO stable

If $|x(t)| < M$,

$$\begin{aligned} |y(t)| &= \left| \int_{-\infty}^{t} e^{-|\tau|} x(-\tau) d\tau \right| \\ &\le \int_{-\infty}^{t} e^{-|\tau|} |x(-\tau)| d\tau \quad \text{(triangle inequality)} \\ &< M \int_{-\infty}^{\infty} e^{-|\tau|} d\tau \\ &= 2M \int_{0}^{\infty} e^{-\tau} d\tau \\ &= 2M \end{aligned}$$

Yes

3a) An LTI system has impulse response

$$h(t) = t^2$$

and input

$$x(t) = \sum_{i=0}^{\infty} \frac{d^i}{dt^i} \delta(t-i).$$

Find the output $y(t)$.

$$y(t) = x(t) * h(t)$$

$$= \sum_{i=0}^{\infty} \delta^{(i)}(t-i) * t^2$$

$$= \sum_{i=0}^{\infty} \left(\delta^{(i)}(t-i) * t^2\right)$$

$$= \sum_{i=0}^{\infty} \frac{d^i}{dt^i}(t-i)^2 \qquad \text{(sifting property)}$$

$$= t^2 + 2(t-1) + 2$$

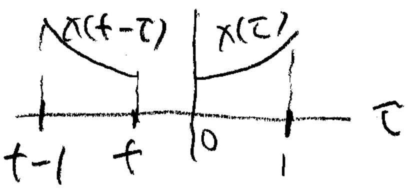
$$= t^2 + 2t$$

b) Let

$$x(t) = \begin{cases} e^t, & 0 < t < 1 \\ 0, & \text{else} \end{cases}.$$

Find $y(t) = x(t) * x(t)$.

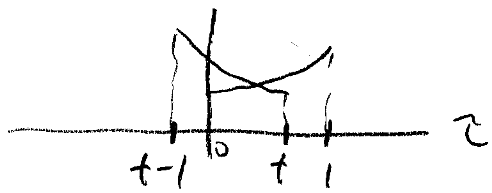
Case I:



$t < 0$

$y(t) = 0$

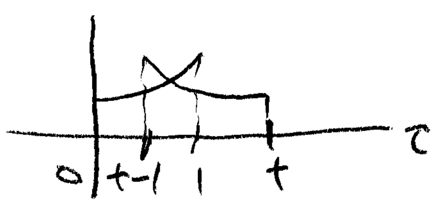
Case II:



$0 < t < 1$

$$y(t) = \int_0^t e^{\tau} e^{t-\tau} d\tau = e^t \int_0^t 1\, d\tau = t e^t$$

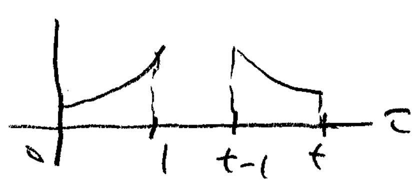
Case III:



$1 < t < 2$

$$y(t) = \int_{t-1}^{1} e^{\tau} e^{t-\tau} d\tau = e^t \int_{t-1}^{1} 1\, d\tau = (2-t)e^t$$

Case IV:



$t > 2$

$y(t) = 0$