Summary of Complex Algebra

Rectangular Form

$$z = x + jy$$

$$\operatorname{Re} z = x$$

$$\operatorname{Im} z = y$$

$$|z| = \sqrt{x^2 + y^2}$$

$$\angle z = \tan^{-1} \frac{y}{x}$$

$$z^* = x - jy$$

$$\frac{1}{z} = \frac{x - jy}{x^2 + y^2}$$

Euler's Formula

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Polar Form

$$z = re^{j\theta}$$

$$\operatorname{Re} z = r\cos\theta$$

$$\operatorname{Im} z = r\sin\theta$$

$$|z| = r$$

$$\angle z = \theta$$

$$z^* = re^{-j\theta}$$

$$\frac{1}{z} = \frac{1}{r}e^{-j\theta}$$

Identities

$$\operatorname{Re} z = \frac{z + z^*}{2}$$

$$\operatorname{Im} z = \frac{z - z^*}{2j}$$

$$|z| = \sqrt{zz^*}$$

$$\frac{1}{z} = \frac{z^*}{|z|^2}$$

$$\operatorname{Re}(z^*) = \operatorname{Re} z$$

$$\operatorname{Im}(z^*) = -\operatorname{Im} z$$

$$|z^*| = |z|$$

$$\angle(z^*) = -\angle z$$

$$(z^*)^* = z$$

$$\frac{1}{z^*} = \left(\frac{1}{z}\right)^*$$

$$\operatorname{Re} \sum z_k = \sum \operatorname{Re} z_k$$

$$\operatorname{Im} \sum z_k = \sum \operatorname{Im} z_k$$

$$\operatorname{Re} \int z(t)\,dt = \int \operatorname{Re} z(t)\,dt$$

$$\operatorname{Im} \int z(t)\,dt = \int \operatorname{Im} z(t)\,dt$$

$$\left(\sum z_k\right)^* = \sum (z_k^*)$$

$$\left(\int z(t)\,dt\right)^* = \int z^*(t)\,dt$$

$$\left|\prod z_k\right| = \prod |z_k|$$

$$\angle\left(\prod z_k\right) = \sum (\angle z_k)$$

$$\left(\prod z_k\right)^* = \prod (z_k^*)$$

Triangle Inequality

$$\left|\sum z_k\right| \le \sum |z_k|$$

$$\left|\int z(t)\,dt\right| \le \int |z(t)|\,dt$$

Exponentials

$$s = \sigma + j\theta$$

$$e^s = e^\sigma e^{j\theta}$$

$$\operatorname{Re}(e^s) = e^\sigma \cos\theta$$

$$\operatorname{Im}(e^s) = e^\sigma \sin\theta$$

$$|e^s| = e^{\operatorname{Re} s}$$

$$\angle e^s = \operatorname{Im} s$$

$$\left|e^{j\theta}\right| = 1$$

$$\angle e^{j\theta} = \theta$$

$$(e^s)^* = e^{(s^*)}$$

$$\frac{1}{e^s} = e^{-s}$$

$$z \neq 0 \implies e^{\ln|z| + j\angle z} = z$$

$$e^{\Sigma s_k} = \prod e^{s_k}$$

$$(e^s)^n = e^{ns}$$