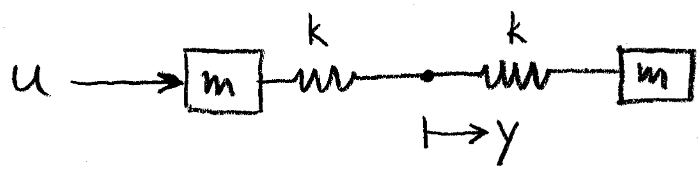
# ECE 332
# Homework #1

1) Consider the spring-mass system shown below. The input $u$ is force, and the output $y$ is the deflection from the equilibrium position. The mass and spring constant are $m = k = 1$.




a) Verify that the system is governed by the differential equation

$$y^{(4)} + 2\ddot{y} = \frac{1}{2}\ddot{u} + u.$$

(Hint: First write Newton's 2nd law for each mass.)

b) Find the eigenvalues of the system.

c) Write the general form of the natural response of the system.

d) Find the transfer function.

e) Find the poles and zeros.

f) Are there any hidden modes?

2) For each of the following rational functions, perform polynomial division of denominator into numerator and write the function as a sum of a polynomial and a strictly proper part.

a) $\frac{s^3+s^2+s+1}{s}$

b) $\frac{s^3+s^2-s-1}{s+1}$

c) $\frac{s^3-s^2+s-1}{s^2+s+1}$

d) $\frac{s^4+4s^3+6s^2+4s+1}{-2s+2}$

3) For each of the following rational functions, use the euclidean algorithm to find the GCD of the numerator and denominator and find the poles and zeros.

a) $\frac{s^3+s^2-2}{s^4+4s^3+8s^2+8s+4}$

b) $\frac{s^3-s^2+2}{s^5+s^4+s^3+\frac{1}{2}s^2+s+9}$

4) In MATLAB, type $n = 2; hw14$ to run the Euclidean algorithm on a pair of randomly chosen polynomials of degree 2. The program returns the greatest common denominator $gcd$ of the two polynomials along with the amount of time $etime$ required to compute $gcd$. For comparison, the program also computes the roots of the polynomials (not displayed) and returns the amount of time $rtime$ required to do so. Increase the value of $n$ and notice how $etime$ and $rtime$ change. Continue to increase $n$ until computation time becomes intolerable. How long does it take to run the Euclidean algorithm in this case?

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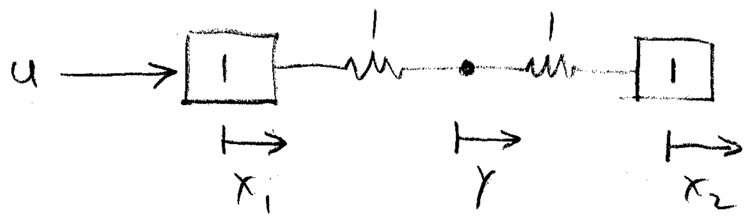
Homework #1

Solutions

1)




a) From Newton's 2nd law,

$$\left.\begin{aligned} \ddot{x}_1 &= u + \tfrac{1}{2}(x_2 - x_1) \\ \ddot{x}_2 &= \tfrac{1}{2}(x_1 - x_2) \\ y &= \tfrac{1}{2}(x_1 + x_2) \end{aligned}\right\}$$

$$x_1^{(4)} = \ddot{u} + \tfrac{1}{2}(\ddot{x}_2 - \ddot{x}_1) = \ddot{u} + \tfrac{1}{2}(x_1 - x_2) - \tfrac{1}{2}u$$

$$x_2^{(4)} = \tfrac{1}{2}(\ddot{x}_1 - \ddot{x}_2) = \tfrac{1}{2}(x_2 - x_1) + \tfrac{1}{2}u$$

$$y^{(4)} + 2\ddot{y} = \tfrac{1}{2}(x_1^{(4)} + x_2^{(4)}) + \ddot{x}_1 + \ddot{x}_2$$

$$= \tfrac{1}{2}\ddot{u} + u$$

b) $\Delta(s) = s^4 + 2s^2 = s^2(s^2+2) = s^2(s+j\sqrt{2})(s-j\sqrt{2})$

eigenvalues : $0, 0, \pm j\sqrt{2}$

c) $y(t) = (A+Bt)e^{0\cdot t} + Ce^{j\sqrt{2}t} + De^{-j\sqrt{2}t}$

Since $\pm j\sqrt{2}$ are complex conjugate, $D = C^*$

$y(t) = A + Bt + 2\mathrm{Re}\,C \cos\sqrt{2}t - 2\mathrm{Im}\,C \sin\sqrt{2}t$

d) $G(s) = \dfrac{\frac{1}{2}s^2+1}{s^4+s^2} = \dfrac{1}{2}\dfrac{s^2+2}{s^2(s^2+2)} = \dfrac{1}{2s^2}$

e) There is a pole $p=0$ with multiplicity 2 and no zero.

f) There are hidden modes $\lambda = \pm j\sqrt{2}$.

2a) By inspection,

$$\frac{s^3+s^2+s+1}{s} = s^2+s+1+\frac{1}{s}$$

b)

$$\begin{array}{r|rrrr}
-1 & 1 & 1 & -1 & -1 \\
 & & -1 & 0 & 1 \\
\hline
 & 1 & 0 & -1 & \boxed{0}
\end{array}$$

$$Q = s^2-1, \quad R = 0$$

$$\frac{s^3+s^2-s-1}{s+1} = s^2-1$$

c)

$$\begin{array}{rr|rrrr}
-1 & -1 & 1 & -1 & 1 & -1 \\
 & & & & -1 & 2 \\
 & & & -1 & 2 & \\
\hline
 & & 1 & -2 & 2 & 1
\end{array}$$

$$\frac{s^3-s^2+s-1}{s^2+s+1} = s-2+\frac{2s+1}{s^2+s+1}$$

d) $-2s+2 = -2(s-1)$

$$
\begin{array}{c|ccccc}
1 & 1 & 4 & 6 & 4 & 1 \\
 & & 1 & 5 & 11 & 15 \\
\hline
 & 1 & 5 & 11 & 15 & \boxed{16}
\end{array}
$$

$$\frac{s^4+4s^3+6s^2+4s+1}{s-1} = s^3+5s^2+11s+15+\frac{16}{s-1}$$

$$\frac{s^4+4s^3+6s^2+4s+1}{-2s+2} = -\frac{1}{2}s^3-\frac{5}{2}s^2-\frac{11}{2}s-\frac{15}{2}+\frac{16}{-2s+2}$$

3a) Apply the euclidean algorithm.

(i) Divide $N = s^3+s^2-2$ into $D = s^4+4s^3+8s^2+8s+4$.

$$
\begin{array}{ccc|ccccc}
-1 & 0 & 2 & 1 & 4 & 8 & 8 & 4 \\
 & & & & & & 2 & 6 \\
 & & & & & 0 & 0 & \\
 & & & & -1 & -3 & & \\
\hline
 & & & 1 & 3 & 5 & 10 & 10
\end{array}
$$

$Q_1 = s+3$, $R_1 = 5s^2+10s+10$

(ii) Divide $\frac{R_1}{5}$ into $N$.

$$\frac{R_1}{5} = s^2+2s+2$$

$$
\begin{array}{cc|cccc}
-2 & -2 & 1 & 1 & 0 & -2 \\
 & & & & -2 & 2 \\
 & & & -2 & 2 & \\
\hline
 & & 1 & -1 & 0 & 0
\end{array}
$$

$$Q_2 = s-1, \quad R_2 = 0$$

$$k=2, \; R_k = 0, \; R_{k-1} = 5s^2+10s+10$$

$$GCD = s^2+2s+2$$

To find the poles and zeros, we must cancel the GCD from N and D.

From step ii), $\frac{N}{GCD} = s-1.$

$\frac{D}{GCD}$:

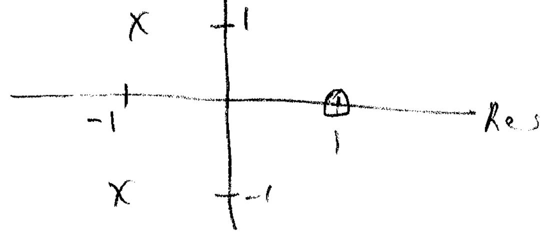
$$\begin{array}{cc|ccc|cc} -2 & -2 & 1 & 4 & 8 & 8 & 4 \\ & & & & -2 & -4 & -4 \\ & & & -2 & -4 & -4 & \\ \hline & & 1 & 2 & 2 & 0 & 0 \end{array}$$

$$\frac{D}{GCD} = s^2+2s+2$$

$$\frac{N}{D} = \frac{s-1}{s^2+2s+2} = \frac{s-1}{(s+1+j)(s+1-j)}$$

zero: $s = 1$

poles: $s = -1 \pm j$



b) Divide $N$ into $D$.

$$
\begin{array}{ccc|cccccc}
1 & 0 & -2 & 1 & 1 & 1 & \frac{1}{2} & 1 & 9 \\
\hline
 & & & & & & -2 & -4 & -6 \\
 & & & & & 0 & 0 & 0 & \\
 & & & & 1 & 2 & 3 & & \\
\hline
 & & & 1 & 2 & 3 & \frac{3}{2} & -3 & 3
\end{array}
$$

$Q_1 = s^2 + 2s + 3, \quad R_1 = \frac{3}{2}s^2 - 3s + 3$

Divide $\frac{R_1}{3/2} = s^2 - 2s + 2$ into $N$.

$$
\begin{array}{cc|cccc}
2 & -2 & 1 & -1 & 0 & 2 \\
\hline
 & & & & -2 & -2 \\
 & & & 2 & 2 & \\
\hline
 & & 1 & 1 & 0 & 0
\end{array}
$$

$Q_2 = s + 1, \quad R_2 = 0$

$GCD = \frac{R_1}{3/2} = s^2 - 2s + 2$

$$\frac{N}{GCD} = s+1$$

$\frac{D}{GCD}$:

$$\begin{array}{cc|cccccc}
2 & -2 & 1 & 1 & 1 & \frac{1}{2} & 1 & 9 \\
 & & & & -2 & -6 & -10 & -9 \\
 & & & 2 & 6 & 10 & 9 & \\
\hline
 & & 1 & 3 & 5 & \frac{9}{2} & 0 & 0
\end{array}$$

$$\frac{D}{GCD} = s^3+3s^2+5s+\frac{9}{2}$$

$$\frac{N}{D} = \frac{s+1}{s^3+3s^2+5s+\frac{9}{2}}$$

zero: $-1$

poles: $-1.63,\ -.69 \pm j1.51$