ECE 332
Homework #2

1) Draw the Nyquist and Bode plots for each of the following rational functions.

a) $\frac{1}{s^3+3s^2+2s}$
b) $\frac{s+1}{s^4+5s^3+6s^2}$
c) $\frac{s^2+1}{s^2-s}$
d) s^3+3s^2+2s

2) Determine whether each of the following is BIBO stable.

a) $\frac{s+1}{s^2-1}$
b) $\frac{s-1}{s^2-1}$
c) $\frac{s+1}{s^2+2s+1}$

3) Determine whether each of the following is BIBO stable.

a) $\frac{s^2-2s+2}{s^4+4s^3+8s^2+8s+4}$
b) $\frac{s^2-2s+2}{s^5+s^4+s^3+\frac{1}{2}s^2+s+9}$

4) For each of the following, determine whether the damping constant is greater than unity.

a) $\frac{s}{s^2+2s}$
b) $\frac{s+2}{s^2+4s+4}$
c) $\frac{s^2-2s+2}{s^5+s^4+s^3+\frac{1}{2}s^2+s+9}$

5) In MATLAB, type $n=2; hw25$ to construct the Routh table for a randomly chosen polynomial of degree 2. The program returns the number of first-column sign changes $rhpcount$ in the table along with the amount of time $htime$ required to compute $rhpcount$. For comparison, the program also computes the roots of the polynomial (not displayed) and returns the amount of time $rtime$ required to do so. Increase the value of n and notice how the ratio changes. Continue to increase n until computation time becomes intolerable. How long does it take to compute first-column sign changes in this case?

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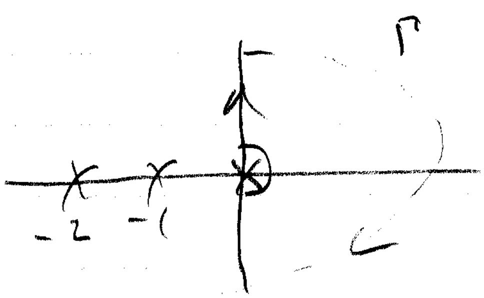
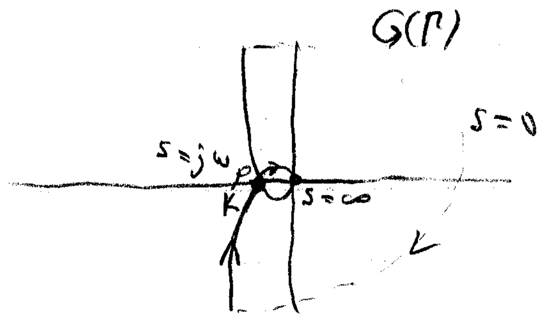
Solutions

(a) Nyquist:

$$G(s) = \frac{1}{s^3+3s^2+2s} = \frac{1}{s(s+1)(s+2)}$$





$$\angle G(j\omega_p) = -90^\circ - \angle j\omega_p+1 - \angle j\omega_p+2$$

$$= -90^\circ - \angle (j\omega_p+1)(j\omega_p+2)$$

$$= -90^\circ - \angle 2+\omega_p^2+j3\omega_p$$

$$= -180^\circ$$

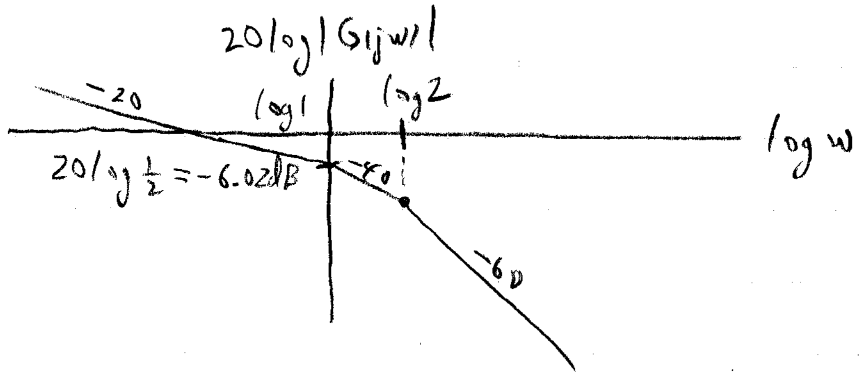
$$\angle 2-\omega_p^2+j3\omega_p = 90^\circ$$

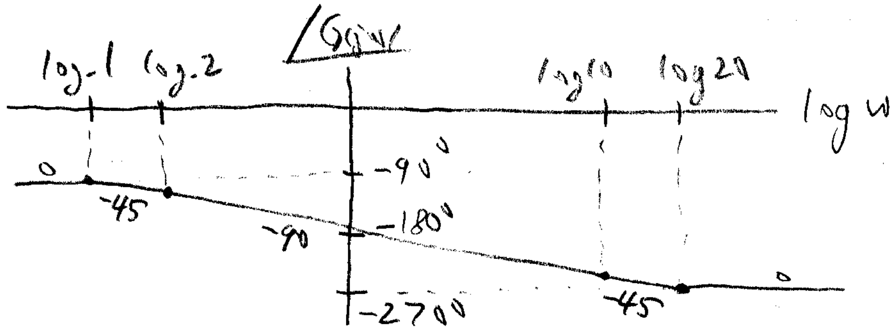
$$2-\omega_p^2 = 0$$

$$\omega_p = \sqrt{2}$$

$$K = G(j\sqrt{2}) = \frac{1}{-j2\sqrt{2} - 3(2) + j2\sqrt{2}} = -\frac{1}{6}$$

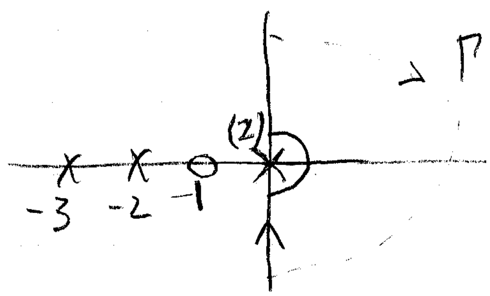
Bode: $$G(s) = \frac{1}{2}\,\frac{1}{s(s+1)\left(\frac{s}{2}+1\right)}$$

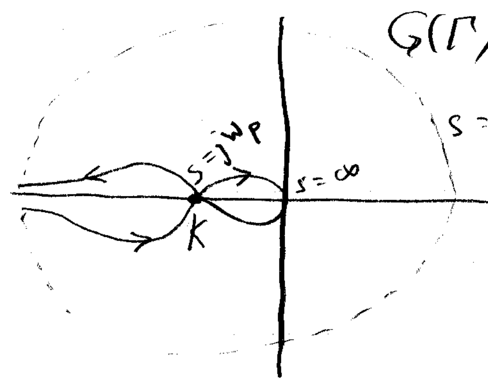




6) Nyquist:

$$G(s) = \frac{s+1}{s^4+5s^3+6s^2} = \frac{s+1}{s^2(s+2)(s+3)}$$





$$\angle G(j\omega_p) = -180^\circ + \angle j\omega_p+1 - \angle j\omega_p+2 - \angle j\omega_p+3$$

$$= -180^\circ + \angle \frac{j\omega_p+1}{6-\omega_p^2+j5\omega_p}$$

$$= -180^\circ + \angle (j\omega_p+1)(6-\omega_p^2-j5\omega_p)$$

$$= -180^\circ + \angle 6-\omega_p^2+5\omega_p^2+j\omega_p(6-\omega_p^2-5)$$

$$= -180^\circ + \angle 6+4\omega_p^2+j\omega_p(1-\omega_p^2)$$

$$= -180^\circ$$

$$\angle\, 6+4\omega_p^2 + j\omega_p(1-\omega_p^2) = 0$$

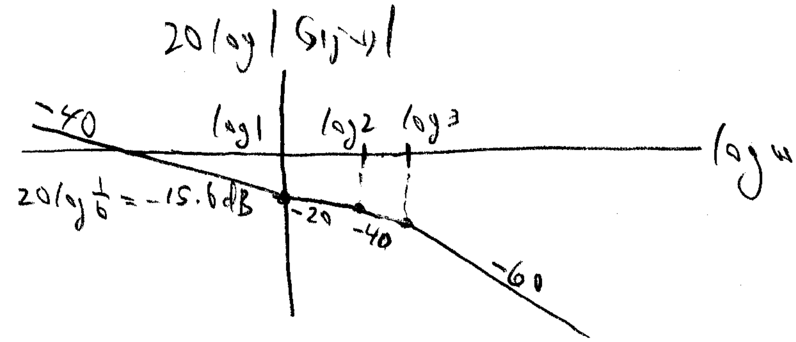
$$\omega_p(1-\omega_p^2) = 0$$

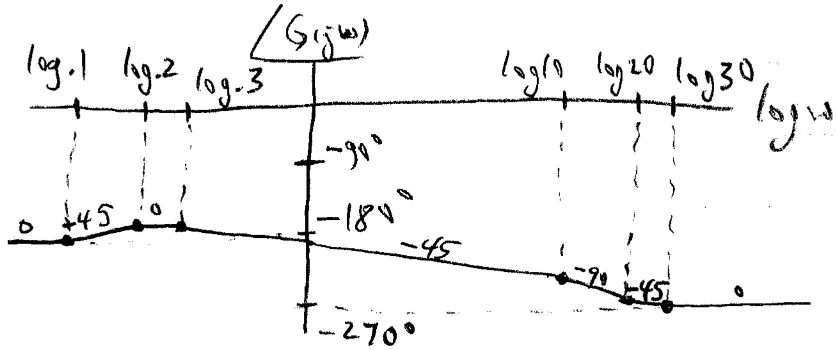
$$\omega_p = 1$$

$$K = G(j) = \frac{1+j}{1-j5-6} = -\frac{1}{5}\,\frac{1+j}{1+j} = -\frac{1}{5}$$

Bode:

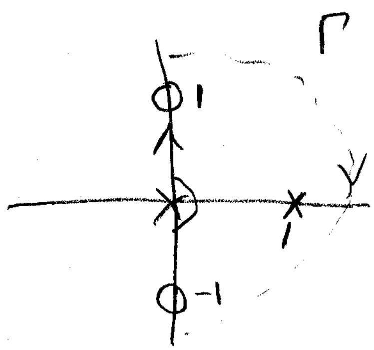
$$G(s) = \frac{1}{6}\,\frac{s+1}{s^2\left(\frac{s}{2}+1\right)\left(\frac{s}{3}+1\right)}$$

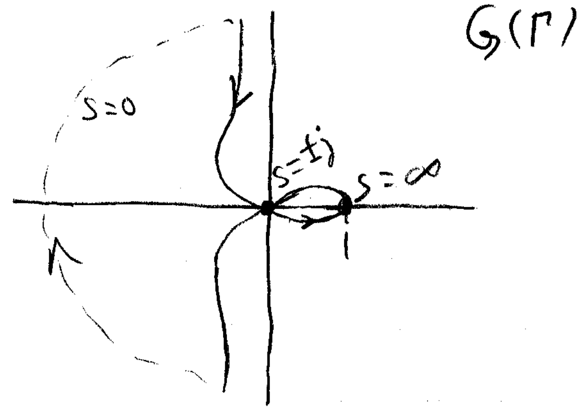




c) Nyquist:

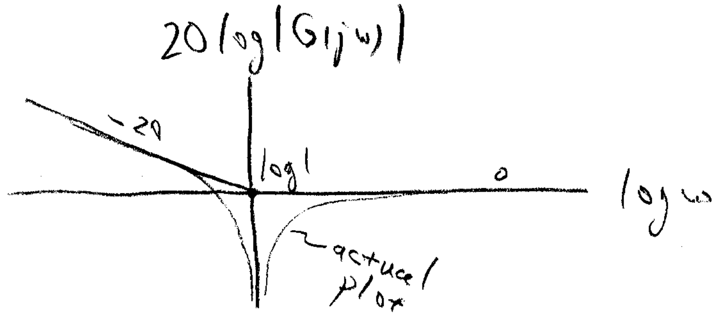
$$G(s) = \frac{s^2+1}{s^2-s} = \frac{(s+j)(s-j)}{s(s-1)}$$

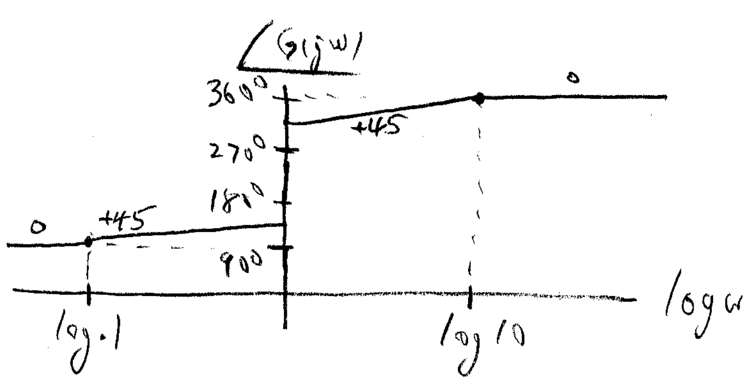




Bode:

$$G(s) = -\frac{s^2+1}{s\left(\frac{s}{-1}+1\right)}$$



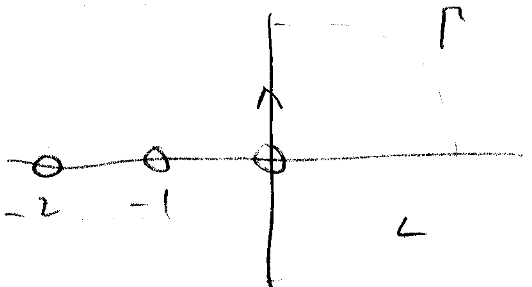


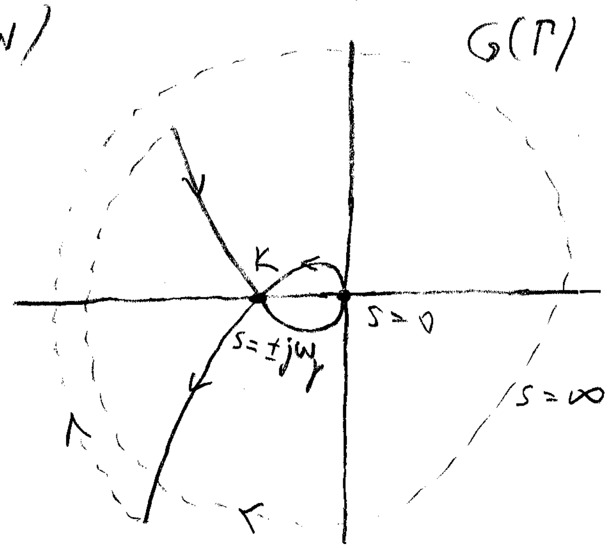
d) Nyquist:

$G(s) = s^3+3s^2+2s = s(s+1)(s+2)$

$G(j\omega) = (j\omega)^3 + 3(j\omega)^2 + 2(j\omega)$

$= -3\omega^2 - j(\omega^3 - 2\omega)$

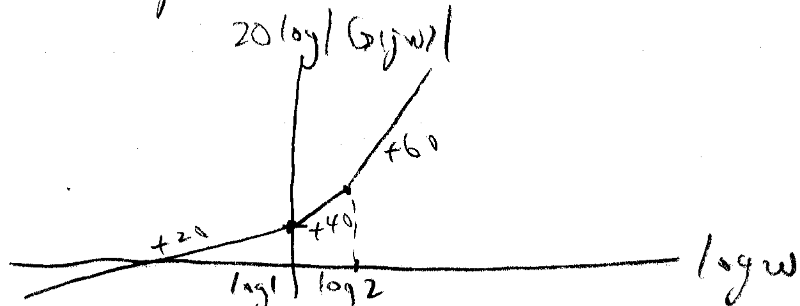


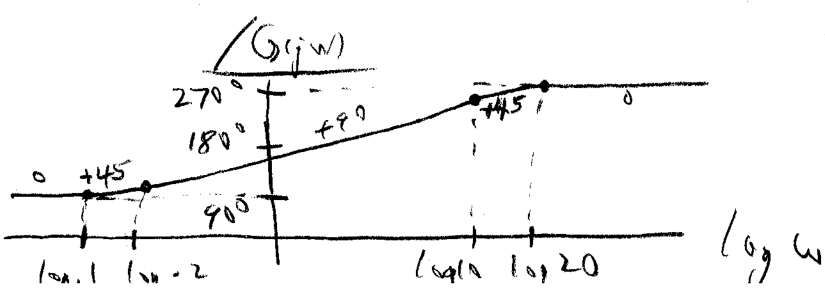


$\angle G(j\omega_p) = 180^\circ \Rightarrow \omega_p^3 - 2\omega_p = 0 \Rightarrow \omega_p = \sqrt{2}$

$K = G(j\sqrt{2}) = -3(\sqrt{2})^2 = -6$

Bode: G(s) in parts a) and d) are reciprocal, so the Bode plots for d) are obtained by simply negating the plots in a).





2a) $\frac{s+1}{s^2-1} = \frac{s+1}{(s-1)(s+1)} = \frac{1}{s-1}$

pole: $+1$ $\Rightarrow$ not BIBO stable

b) $\frac{s-1}{s^2-1} = \frac{s-1}{(s-1)(s+1)} = \frac{1}{s+1}$

pole: -1 $\Rightarrow$ BIBO stable

c) $\frac{s+1}{s^2+2s+1} = \frac{s+1}{(s+1)^2} = \frac{1}{s+1}$

pole: -1 $\Rightarrow$ BIBO stable

3a) We need to apply the euclidean algorithm in order to extract a GCD.

Divide N into D.

$$
\begin{array}{cc|ccc|cc}
2 & -2 & 1 & 4 & 8 & 8 & 4 \\
 & & & & -2 & -12 & -36 \\
 & & & 2 & 12 & 36 & \\
\hline
 & & 1 & 6 & 18 & 32 & -32
\end{array}
$$

$Q_1 = s^2 + 6s + 18$, $R_1 = 32s - 32$

Divide $\frac{R_1}{32}$ into N.

$$
\begin{array}{c|cc|c}
1 & 1 & -2 & 2 \\
 & & 1 & -1 \\
\hline
 & 1 & -1 & 1
\end{array}
$$

$Q_2 = s - 1$, $R_2 = 1$

N, D are coprime.

G BIBO stable $\iff$ D Hurwitz.

Construct the Routh table for D.

$$
\text{no sign changes}\left\{
\begin{array}{ccc}
1 & 8 & 4 \\
4 & 8 & 0 \\
6 & 4 & \\
\frac{16}{3} & 0 & \\
4 & &
\end{array}
\right.
$$

G is BIBO stable.

b) euclidean algorithm:

$$
\begin{array}{cc|cccccc}
2 & -2 & 1 & 1 & 1 & \frac{1}{2} & 1 & 9 \\
 & & & & -2 & -6 & -10 & -9 \\
 & & & 2 & 6 & 10 & 9 & \\
\hline
 & & 1 & 3 & 5 & \frac{9}{2} & 0 & 0
\end{array}
$$

$Q_1 = s^3 + 3s^2 + 5s + \frac{9}{2}, \quad R_1 = 0$

$GCD = N = s^2 - 2s + 2$

$\frac{D}{GCD} = Q_1$

We need to apply Routh-Hurwitz to Q_1.

$$\mathcal{H} = \begin{bmatrix} 3 & \frac{9}{2} & 0 \\ 1 & 5 & 0 \\ 0 & 3 & \frac{9}{2} \end{bmatrix}$$

$$m_2 = \begin{vmatrix} 3 & \frac{9}{2} \\ 1 & 5 \end{vmatrix} = \frac{21}{2}$$

Coefficients of $Q_1 > 0$ and $m_2 > 0$

$\Rightarrow$ G is BIBO stable.

4a) $\frac{s}{s^2+2s} = \frac{s}{s(s+2)} = \frac{1}{s+2}$

Let $D(s) = s+2$.

$D(s-1) = (s-1)+2 = s+1$ (Hurwitz)

$\sigma > 1$

b) $\frac{s+2}{s^2+4s+4} = \frac{s+2}{(s+2)^2} = \frac{1}{s+2}$

Same denominator as in a) $\Rightarrow$ $\sigma > 1$.

c) From 7b), $G(s) = \frac{1}{s^3+3s^2+5s+\frac{9}{2}}$.

$D(s-1) = (s-1)^3+3(s-1)^2+5(s-1)+\frac{9}{2}$

$= s^3+2s+\frac{3}{2}$

Since the s^2 term vanishes, $D(s-1)$ is not Hurwitz.

$\sigma \not> 1$