ECE 332
Homework #3

1) Design a second-order transfer function $H_d(s)$ to meet all of the following specifications. Choose ω_n as small as possible.

a) $e_{ss0} = 0$
b) $M_r \leq 1.3$
c) $M_p \leq 1.3$
d) $e_{ss1} \leq .8$ s
e) $\omega_b \geq 1.5$ rad/s
f) $\omega_r \geq .7$ rad/s
g) $T_r \leq 2.3$ s
h) $T_p \leq 3.5$ s
Find the poles of the system.

2) Let

$$H_R(s) = H_d(s)\frac{R(s+R)}{s^2 + .05Rs + R^2},$$

where H_d was found in 1).

a) Find the poles and zeros of H_R for $R = 5$. What is the damping factor ξ for the high-frequency poles?
b) Use MATLAB to find the smallest value of R so that

i) $20\log|H_R(j\omega)| < -40$ dB and $20\log|H_d(j\omega)| < -40$ dB for all ω such that

$$|20\log|H_R(j\omega)| - 20\log|H_d(j\omega)|| > 1\ dB$$

ii) $|y_d(t) - y_R(t)| \leq .01$,

where y_d and y_R are the step responses corresponding to H_d and H_R. To do this, enter "$xi = \xi; wn = \omega_n; R = 5; hw32$", where ξ and ω_n are the values of damping ratio and natural frequency chosen in part 1). MATLAB will display the frequency and step responses for H_d and H_5. Increase R until i) and ii) are met. Print the final plots and identify which curves correspond to each transfer function.

c) Find the poles and zeros of H_R for the value of R selected in b).

3) Consider each of the following all-pass transfer functions H_a.

i) $\frac{s-1}{s+1}$
ii) $\frac{s^2-\sqrt{2}s+1}{s^2+\sqrt{2}s+1}$
iii) $\frac{(s-1)(s^2-\sqrt{2}s+1)}{(s+1)(s^2+\sqrt{2}s+1)}$

For each case, let $H = H_d H_a$,where H_d was obtained in problem 1).

a) Find the poles and zeros of H.
b) Using MATLAB, draw Bode plots for H_d and H. Also plot $y_d(t)$ and $(-1)^n y(t)$, where y_d and y are the step responses corresponding to H_d and H, and n is the order of H_a. For case i), this is done by entering "$xi = \xi; wn = \omega_n; hw33i$". Compare the responses of the minimum-phase and the non-minimum-phase systems. Print the graphs and identify which correspond to H_d and H.

Repeat a) and b) for cases ii) and iii) using the commands "$xi = \xi; wn = \omega_n; hw33ii$" and "$xi = \xi; wn = \omega_n; hw33iii$".

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Solutions

1) First note that a) and b) imply that the system is type 1. This requires

$$A = \omega_n^2.$$

Apply the 2nd-order formulas:

$$e_{ss1} = 2\frac{\xi}{\omega_n}$$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$M_p = 1 + e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}$$

$$\omega_b = \omega_n\sqrt{1-2\xi^2+2\sqrt{1-\xi^2+\xi^4}}$$

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

$$T_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}}$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

The formulas for M_r and M_p involve only ξ, so we will examine them first. From c),

$$\frac{1}{2\xi\sqrt{1-\xi^2}} \leq 1.3$$

$$\xi\sqrt{1-\xi^2} \geq \frac{1}{2.6}$$

$$\xi^2(1-\xi^2) \geq \frac{1}{2.6^2}$$

$$\xi^4 - \xi^2 + \frac{1}{2.6^2} \leq 0$$

$$.43 \leq \xi \leq .90$$

But peaking in the frequency response only occurs for $\xi < \frac{1}{\sqrt{2}}$, so any $\xi \geq .43$ suffices. Substituting $\xi = .43$ in the formula for M_p yields

$$M_p = 1.22,$$

so the M_p specification is met. Inspection of the remaining formulas shows that adopting the minimum ξ also minimizes ω_n.

$$\omega_n = \frac{2\xi}{e_{ss1}} \geq \frac{.86}{.8} = 1.08$$

$$\omega_n = \frac{\omega_b}{\sqrt{1-2\xi^2+2\sqrt{1-\xi^2-\xi^4}}} \geq \frac{1.5}{1.55} = .97$$

$$\omega_n = \frac{\omega_r}{\sqrt{1-2\xi^2}} \geq \frac{.7}{.79} = .88$$

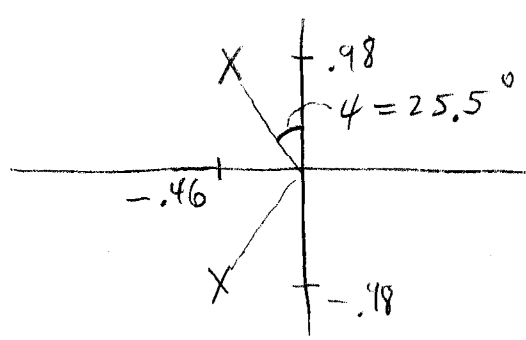
$$\omega_n = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{T_r\sqrt{1-\xi^2}} \geq \frac{2.23}{2.3} = .97$$

$$\omega_n = \frac{\pi}{T_p\sqrt{1-\xi^2}} \geq \frac{3.47}{3.5} = .99$$

Set $\omega_n = 1.08$ rad/sec.

The system has poles

$$p_{1,2} = -\xi\omega_n \pm j\,\omega_n\sqrt{1-\xi^2} = -.46 \pm j.98.$$

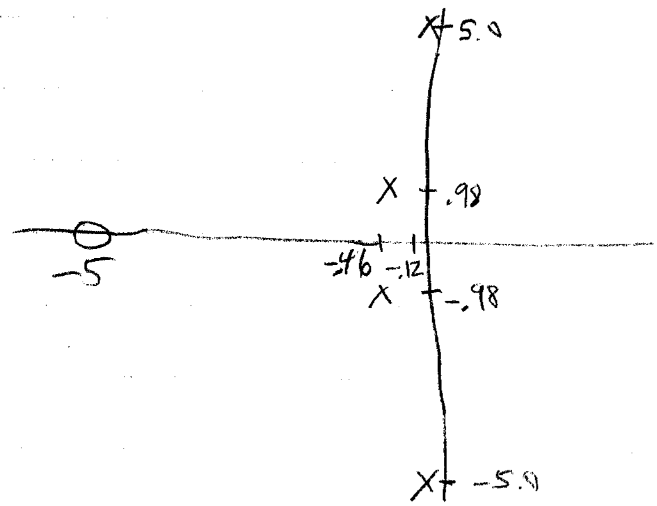


$$H_d(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{1.17}{s^2+.93s+1.17}$$

2a) $$H_5(s) = \frac{1.17}{s^2+.93s+1.17} \cdot \frac{5(s+5)}{s^2+.25s+25}$$

zero: -5

poles: $-.46 \pm j.98, \; -.12 \pm j5.0$



For the high-frequency poles,

$$2\xi\omega_n = .25, \quad \omega_n^2 = 25$$

$$\xi = \frac{.25}{2\omega_n} = \frac{.25}{2\sqrt{25}} = .025$$

b) For $R = 60$, the plots $|H_d(j\omega)|$ and $|H_R(j\omega)|$ are in close agreement for $\omega \le 10$ rad/sec. For $\omega > 10$ rad/sec,

$$20 \log |H_d(j\omega)| < -40\,dB$$

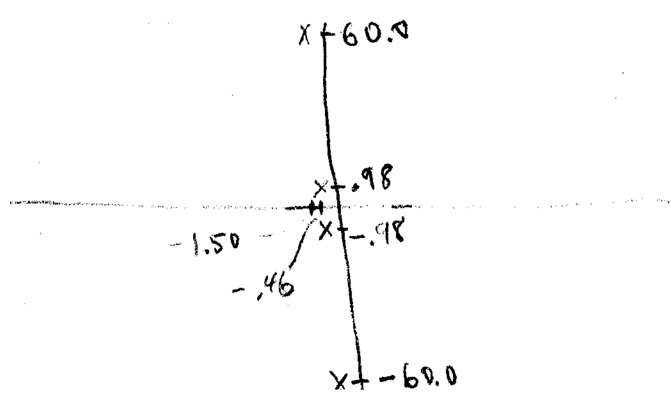
$$20 \log |H_{55}(j\omega)| < -40\,dB,$$

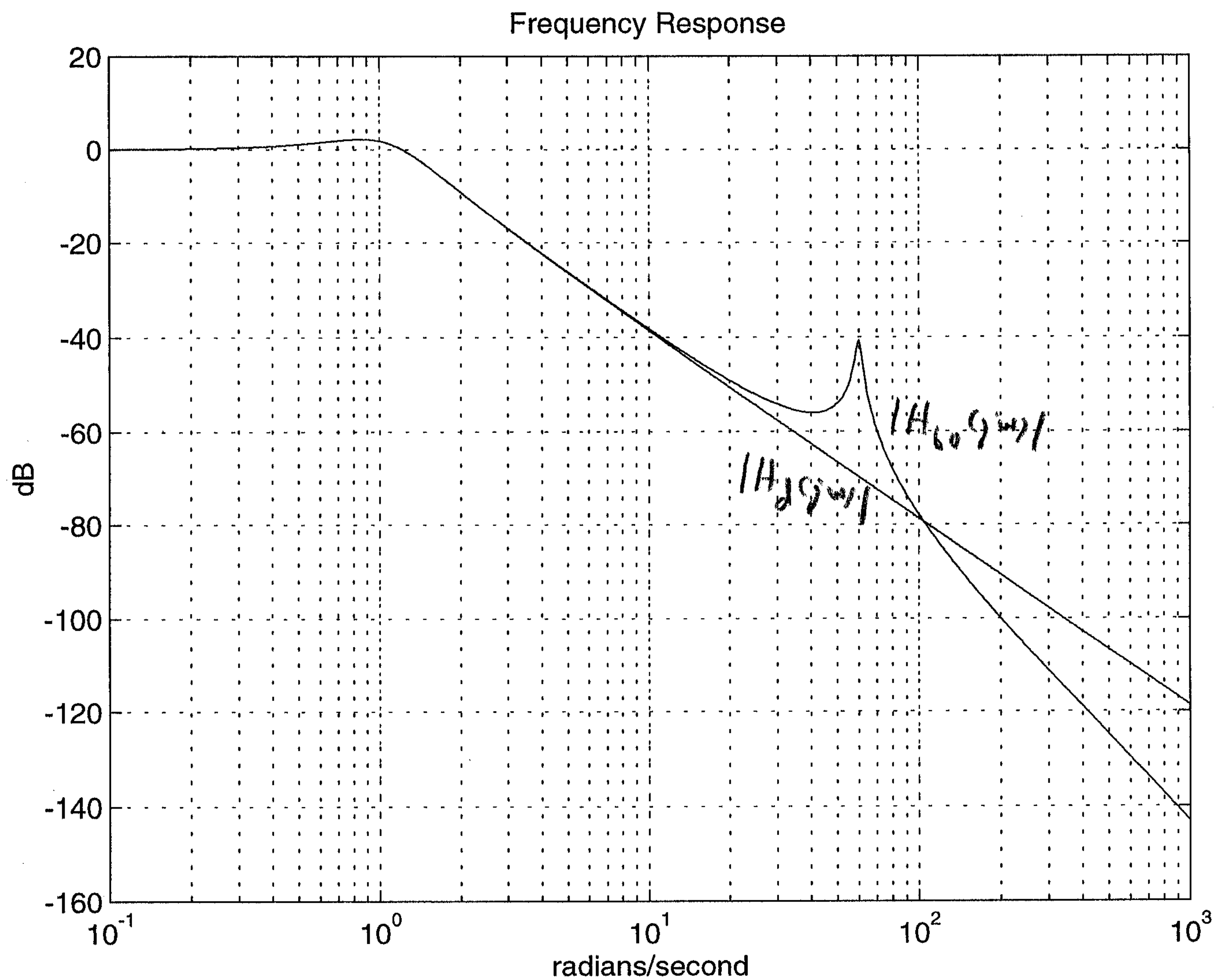
For $k = 60$, $y_d(t)$ and $y_k(t)$ are extremely close.

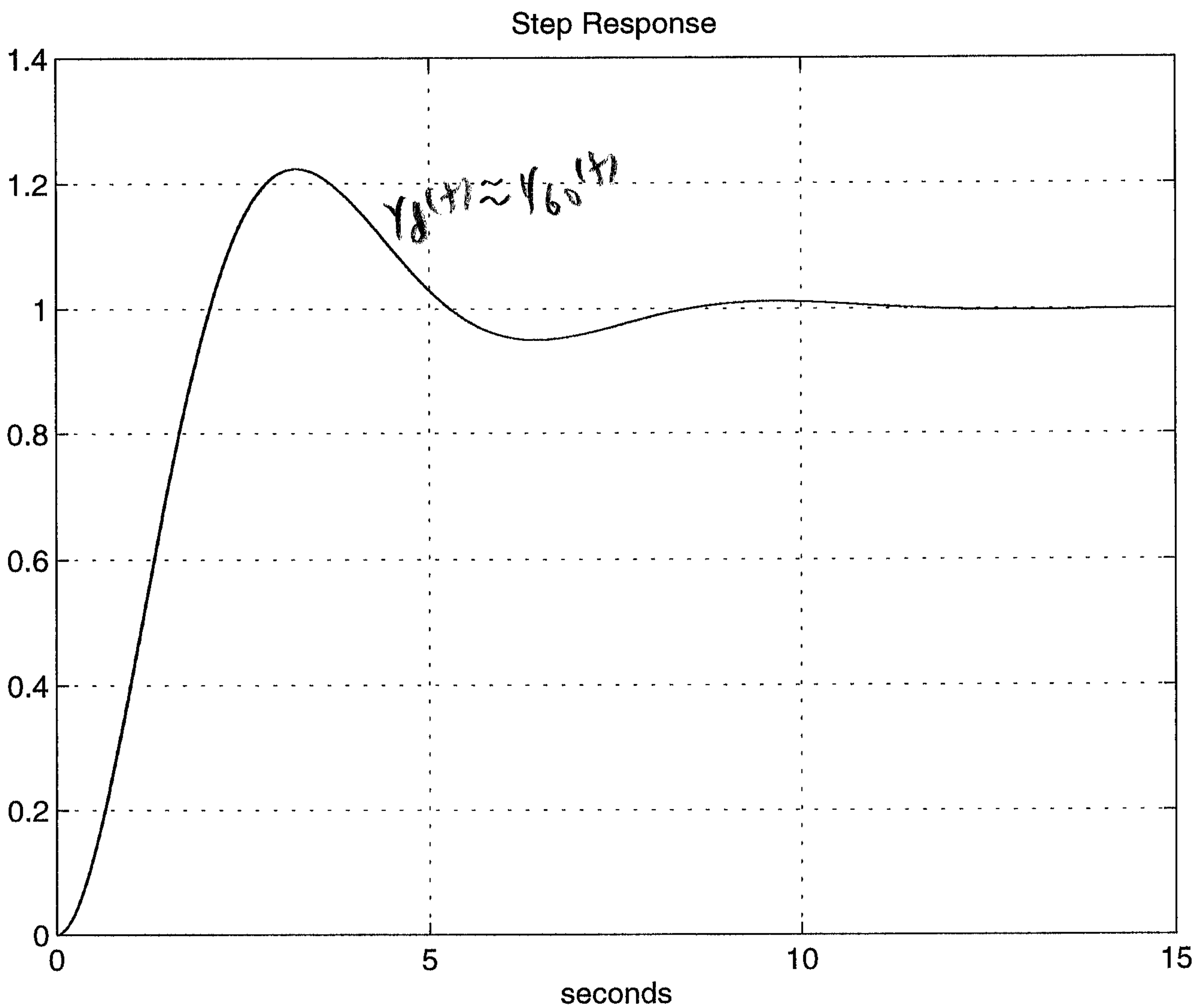
$$H_{60}(s) = \frac{70.0(s+60)}{(s^2+.93s+1.17)(s^2+3.00s+3600)}$$

c) zeros: -60

poles: $-.46 \pm j.98$, $-1.50 \pm j60.0$

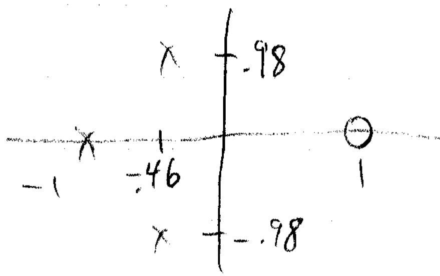



Frequency Response
dB
radians/second
20
0
-20
-40
-60
-80
-100
-120
-140
-160
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
$|H_{60}(j\omega)|$
$|H_d(j\omega)|$


Step Response
1.4
1.2
1
0.8
0.6
0.4
0.2
0
5
10
15
seconds
$y_d(t) \approx y_{60}(t)$

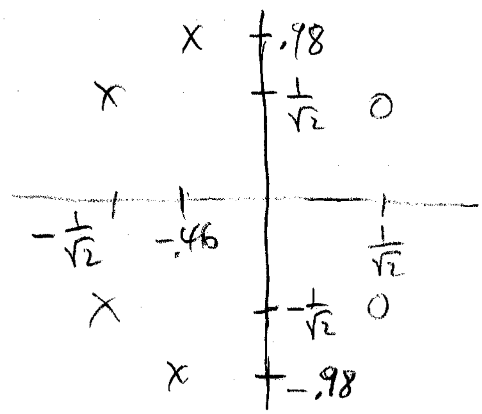
3 i) zeros: 1

poles: $-.46 \pm j.98$, -1



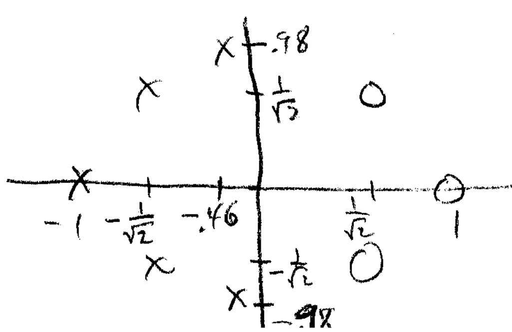
ii) zeros: $\frac{1}{\sqrt{2}}(1 \pm j)$

poles: $-.46 \pm j.98$, $\frac{1}{\sqrt{2}}(-1 \pm j)$



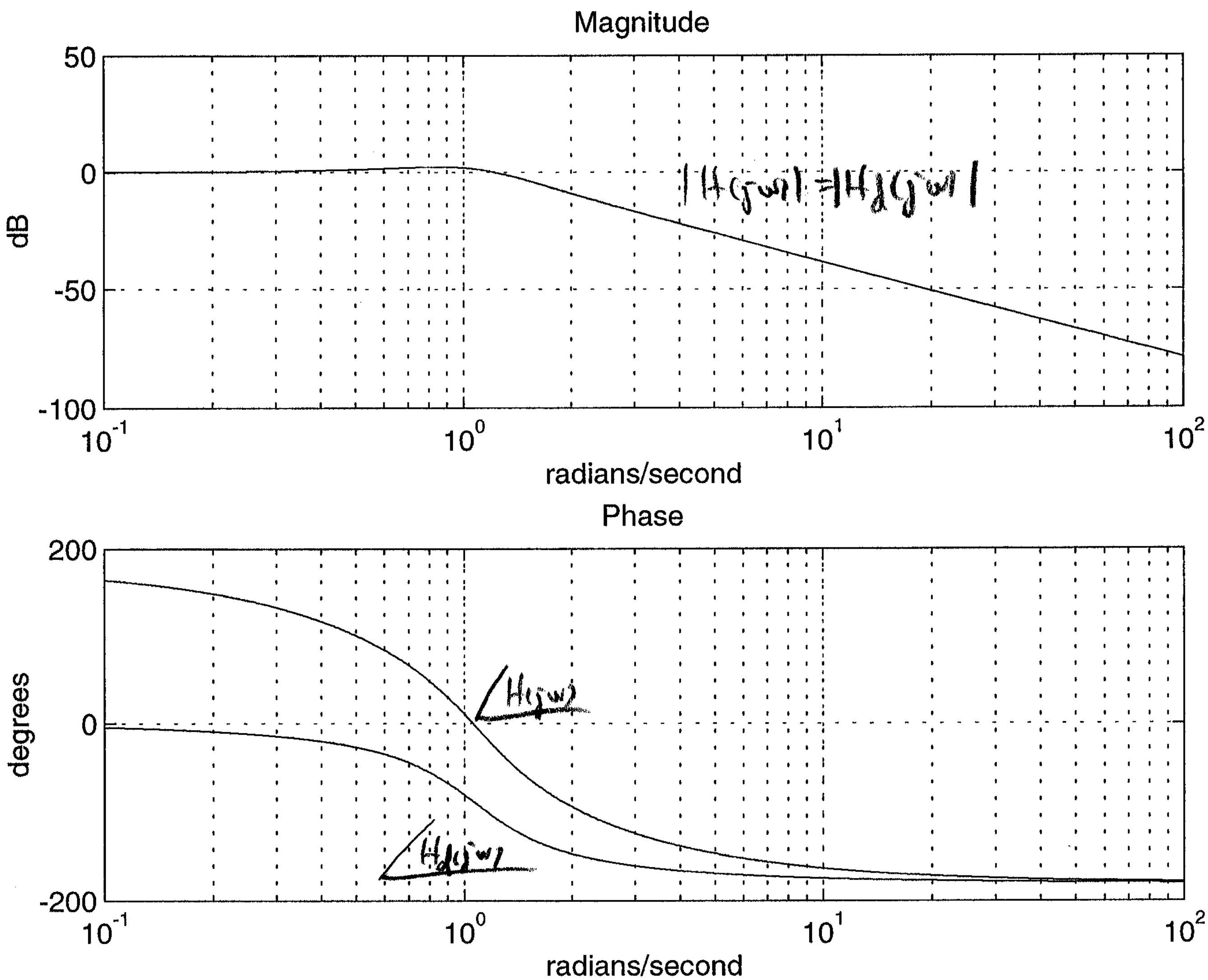
iii) zeros: 1, $\frac{1}{\sqrt{2}}(1 \pm j)$

poles: $-.46 \pm j.98$, -1, $\frac{1}{\sqrt{2}}(-1 \pm j)$

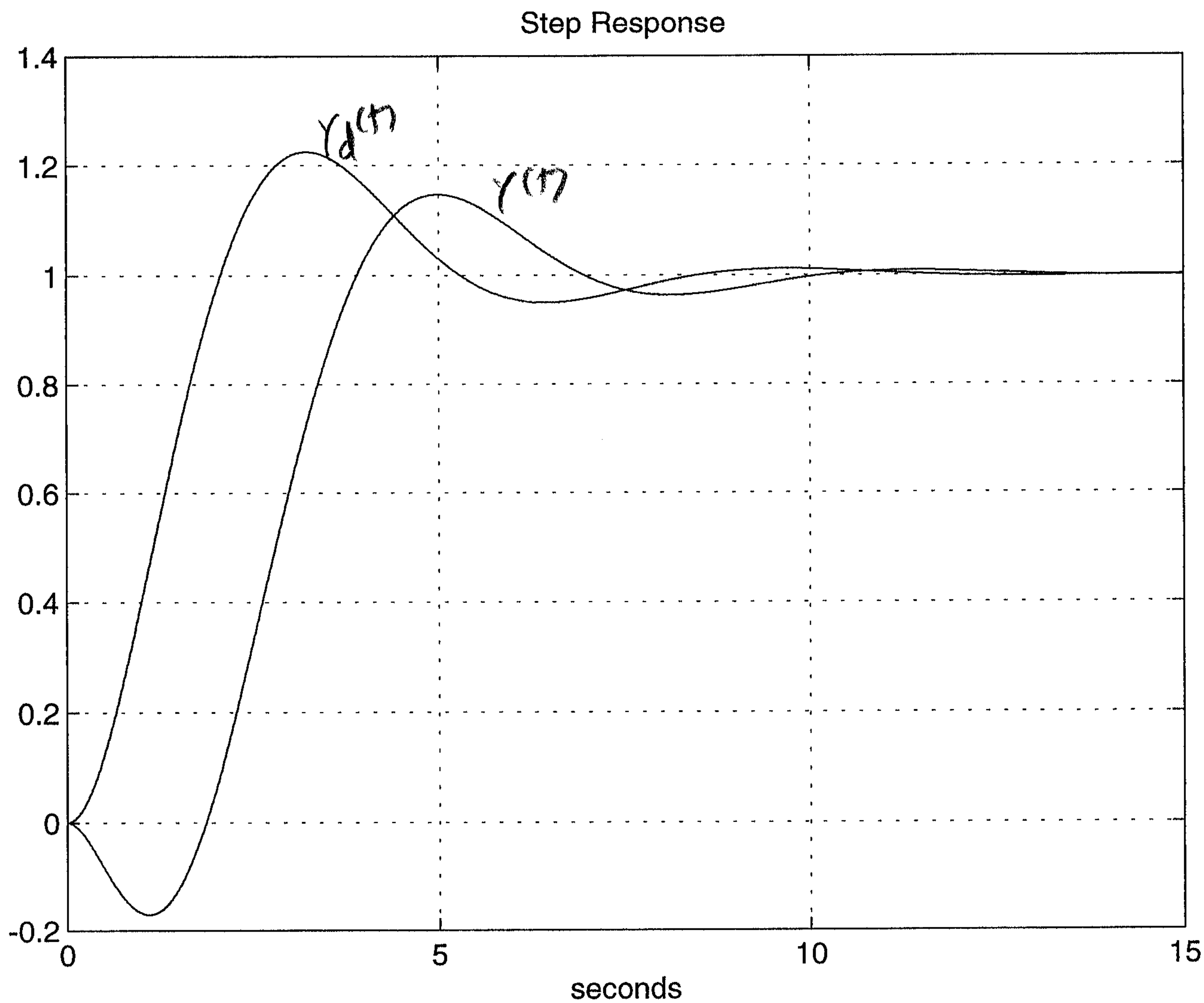


i) H_d has minimum phase.

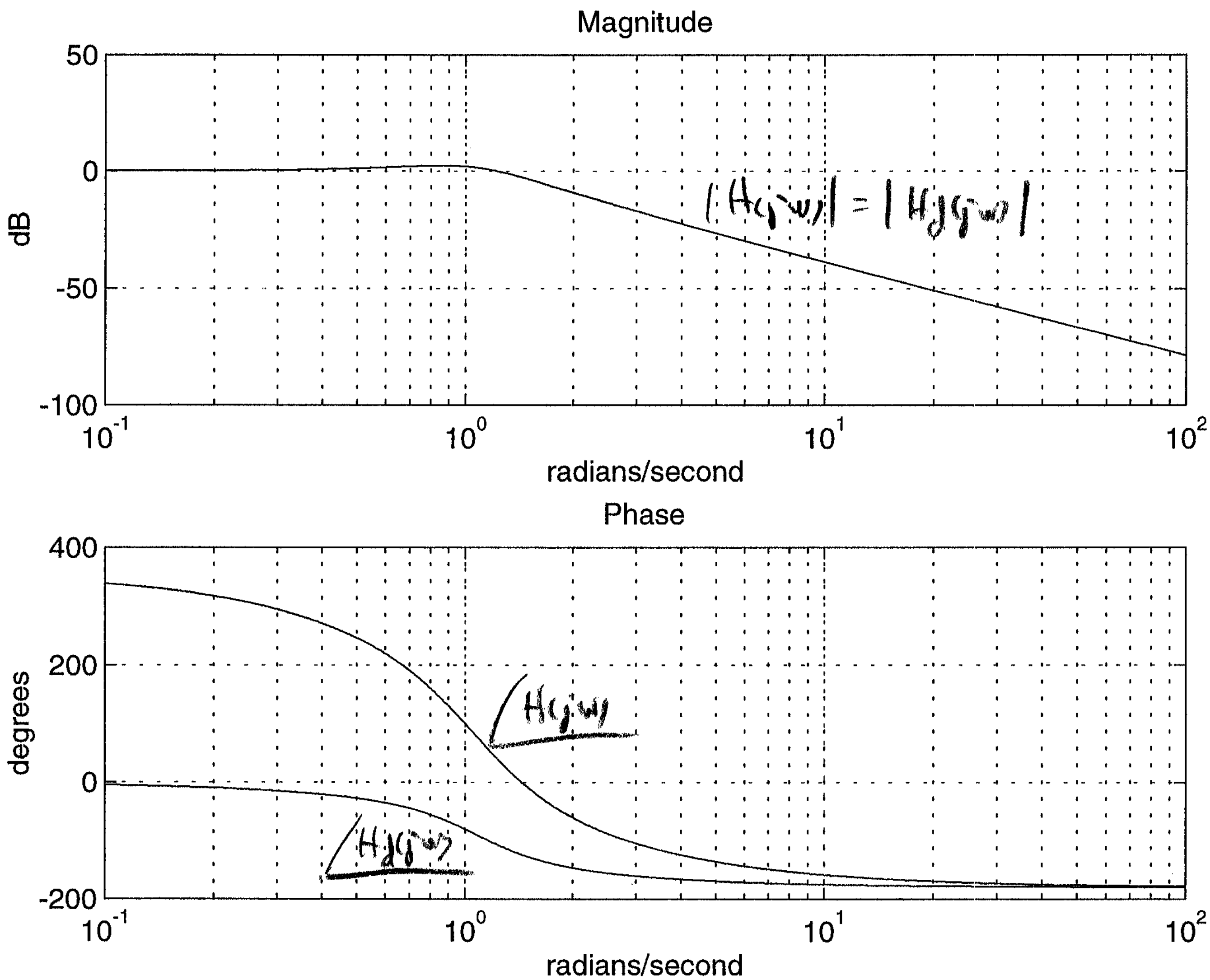
H has non-minimum phase.



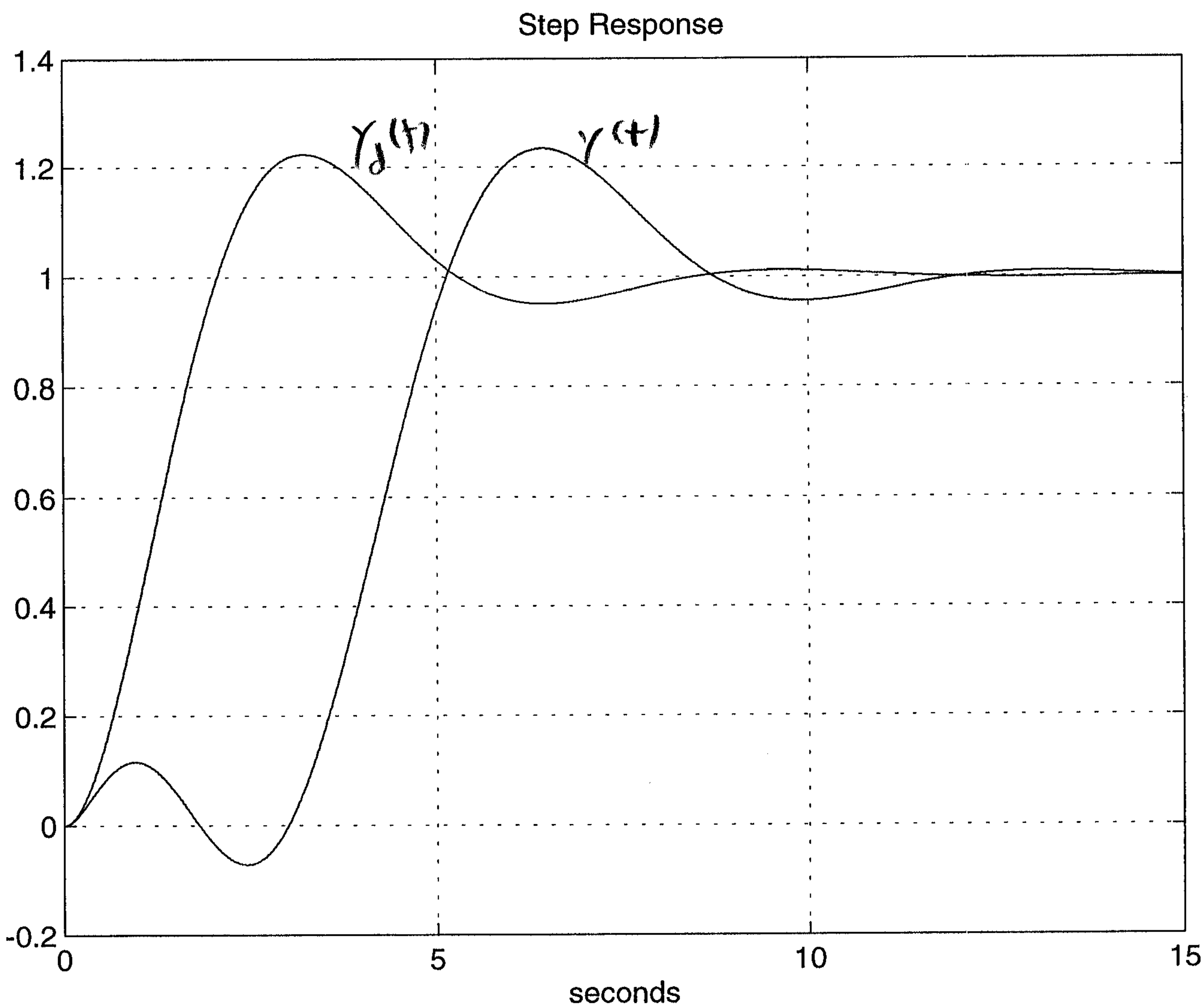
i)



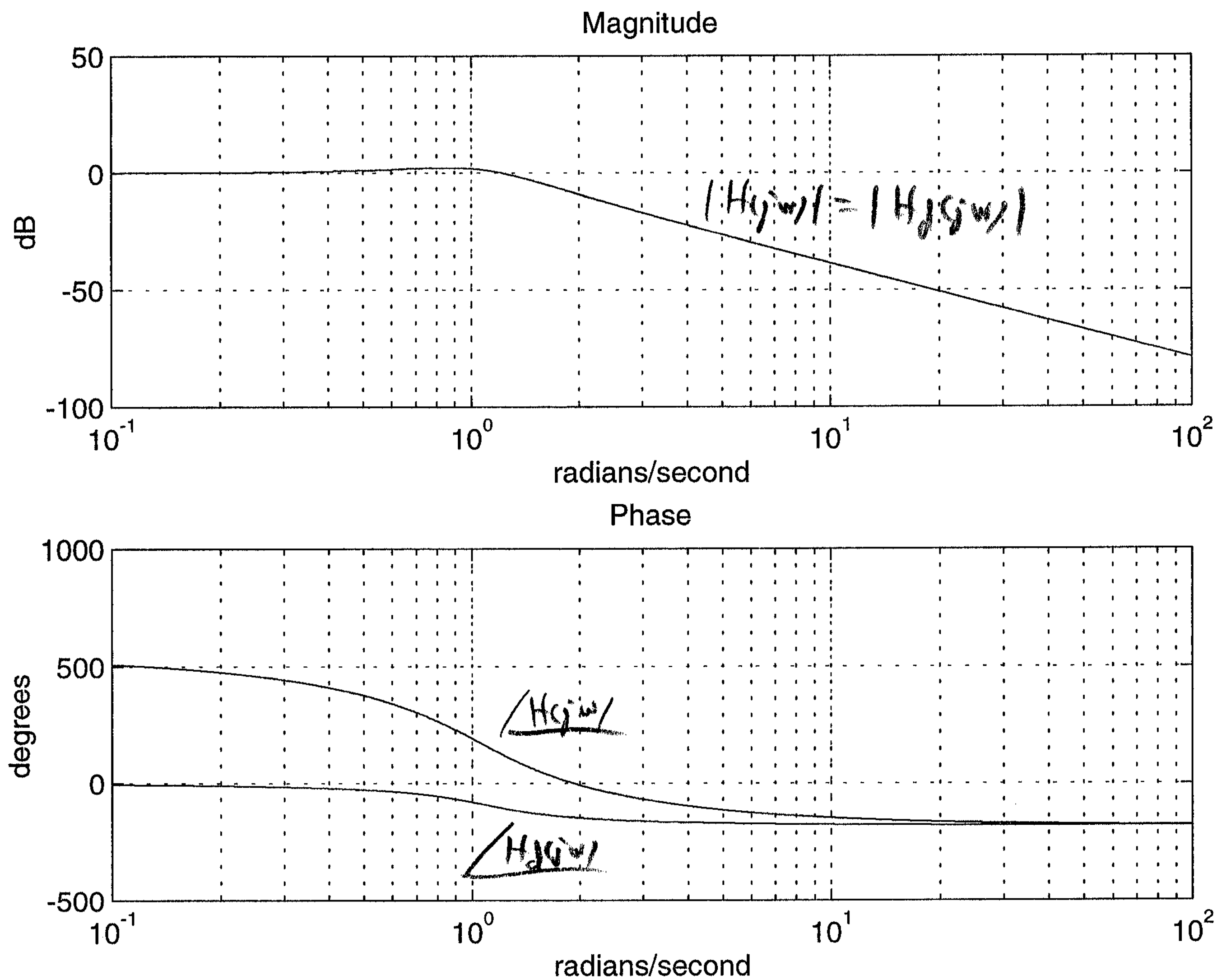
ii)


Magnitude
50
0
-50
-100
dB
10^{-1}
10^{0}
10^{1}
10^{2}
radians/second
$|H(j\omega)| = |H_d(j\omega)|$
Phase
400
200
0
-200
degrees
$\angle H(j\omega)$
$\angle H_d(j\omega)$
10^{-1}
10^{0}
10^{1}
10^{2}
radians/second

ii)



iii)



Magnitude
dB
radians/second
Phase
degrees
radians/second
|H(jw)| = |H_d(jw)|
∠H(jw)
∠H_d(jw)

iii)