ECE 332
Homework #4

1) For each of the following plants $G(s)$, find the closed-loop transfer function $H(s)$ under unity feedback. Using Routh-Hurwitz methods, determine the number of RHP poles in $H(s)$. If $H(s)$ is BIBO stable, find its type number q and the steady-state error e_{ssq} to an order q singularity function.

a) $\frac{20}{(s+1)^3}$
b) $\frac{s+2}{(s-1)(s+1)}$
c) $\frac{1}{(s+1)(s^2+1)}$

2) For each $G(s)$ in problem 1), draw the Nyquist plot and determine the number of RHP poles in $H(s)$. Compare your answers to problem 1).

3) For each $G(s)$ in problem 1), draw the Bode plots and determine the number of RHP poles in $H(s)$. Compare your answers to problem 1).

4) Let

$$G(s) = \frac{10^7(s+10)^5}{s(s+1)^3(s+50)^5}.$$

In MATLAB, type "$hw44$" to display the Bode plots of $G(s)$ as well as the frequency and step response of the closed-loop system under unity feedback.

a) Print the graphs and find all crossover frequencies ω_p and ω_g. Verify that $H(s)$ is BIBO stable.
b) Find the type number q and steady-state error e_{ssq} corresponding to $H(s)$.
c) From the graphs, find ϕ_p, K_g, ω_b, ω_r, M_r, T_r, T_p, and M_p.
d) Compare ω_g and ω_b.

5) Design a type 1 second-order system $H(s)$ to meet the specifications

i) $\omega_g \geq 10\ rad/s$
ii) $\phi_p \geq 60°$

Using the resulting values of ξ and ω_n, type "$xi = \xi; wn = \omega_n; hw45$" in MATLAB to display the Bode plots for the corresponding $G(s)$. Verify that your design actually achieves specifications i) and ii).

6) Using your values of ξ and ω_n from problem 5), type "$xi = \xi; wn = \omega_n; hw46$" in MATLAB to display $|S(j\omega)|$, where S is the sensitivity function corresponding to H. What is the maximum value of $|S(j\omega)|$ and at what frequency does the maximum occur?

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Solutions

(a) $H(s) = \dfrac{G(s)}{1+G(s)} = \dfrac{\frac{20}{s^3+3s^2+3s+1}}{1+\frac{20}{s^3+3s^2+3s+1}} = \dfrac{20}{s^3+3s^2+3s+21}$

Routh-table:

1	3	0
3	21	0
−4	0	
21		

no imaginary pole ; 2 open RHP poles

b) $H(s) = \dfrac{\frac{s+2}{s^2-1}}{1+\frac{s+2}{s^2-1}} = \dfrac{s+2}{s^2+s+1}$ BIBO stable

$G(s)$ has no pole at 0 $\Rightarrow$ $q=0$

$$e_{ss0} = \frac{1}{1+G(0)} = \frac{1}{1-2} = -1$$

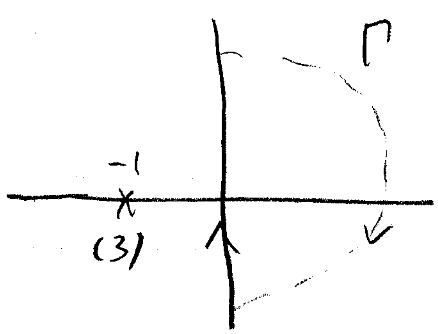
c) $H(s) = \dfrac{\frac{1}{s^3+s^2+s+1}}{1+\frac{1}{s^3+s^2+s+1}} = \dfrac{1}{s^3+s^2+s+2}$

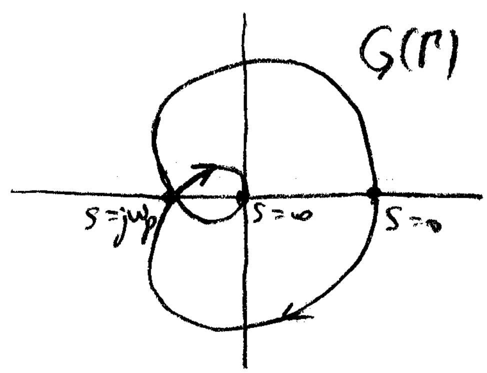
Routh table:

1	1	0
1	2	0
-1	0	
2		

no imaginary pole; 2 open RHP poles

2a)





$p = 0$

$$\angle G(j\omega_p) = -3\angle j\omega+1 = -180^\circ$$

$$\angle j\omega+1 = 60^\circ$$

$$\omega_p = \tan 60^\circ = \sqrt{3}$$

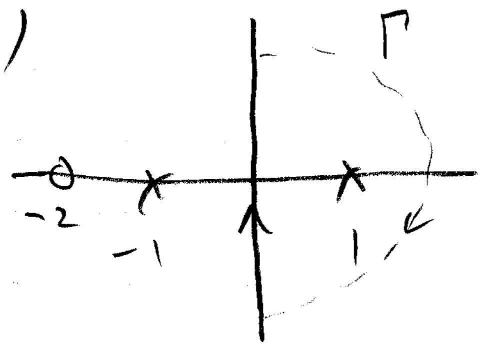
$$|G(j\sqrt{3})| = \frac{20}{|j\sqrt{3}+1|^3} = \frac{20}{2^3} = \frac{5}{2} > 1$$

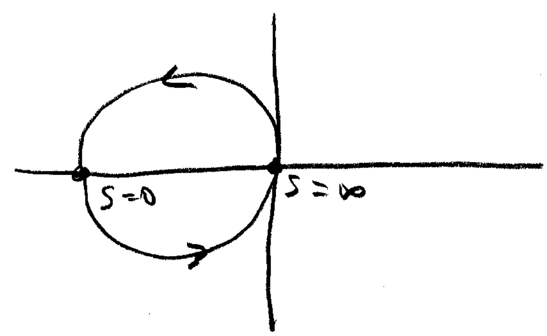
$$\gamma = -2$$

$$p - \gamma = 2$$

H has no imaginary pole and 2 open RHP poles.

b)



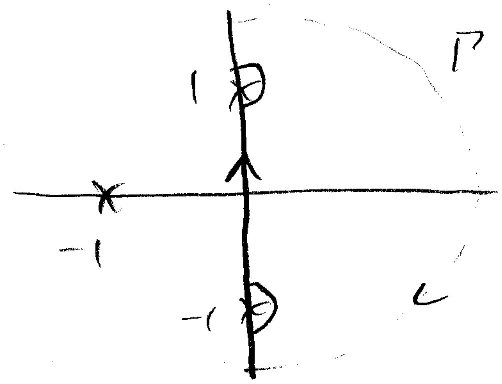


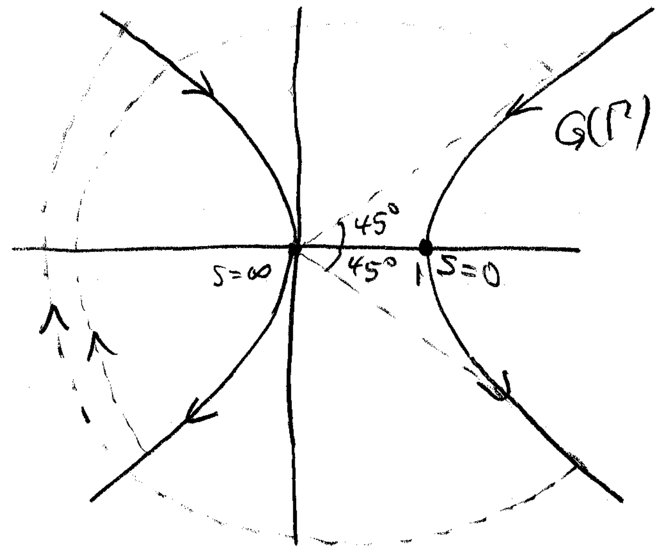
$p = 1$

$|G(0)| = 2 \Rightarrow \gamma = 1$

$$p - \gamma = 0 \Rightarrow \text{BIBO stable}$$

c)



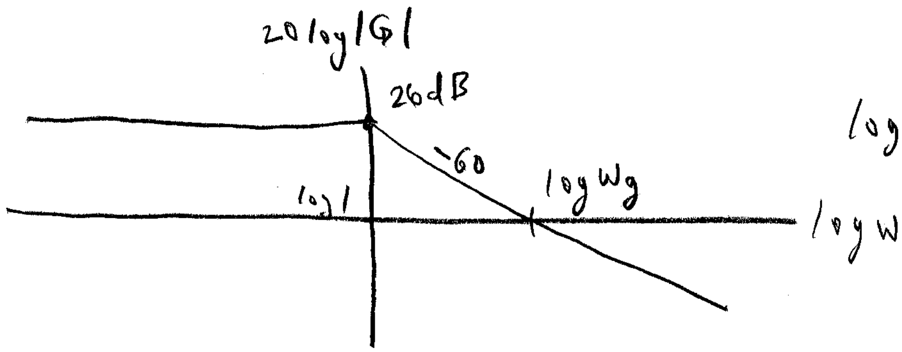


$p = 0$ $\qquad\qquad\qquad$ $\gamma = -2$

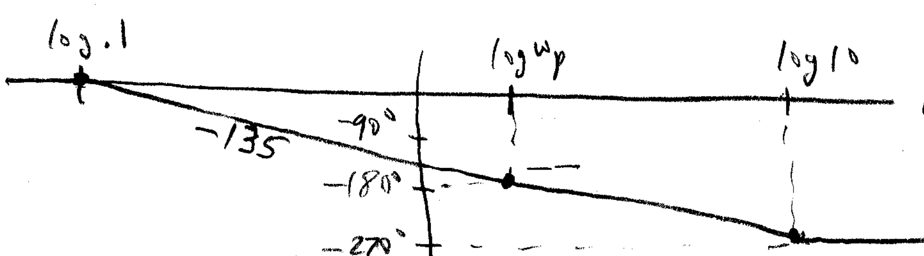
$$p - \gamma = 2$$

H has no imaginary pole and 2 open RHP poles.

3a) $G(s) = 20 \dfrac{1}{(s+1)^3}$ $\qquad$ $(p = 0)$



$\log \omega_g \approx \log 1 + \frac{26}{60} = .43$



$\log \omega_p \approx \log .1 + \frac{180}{135}$

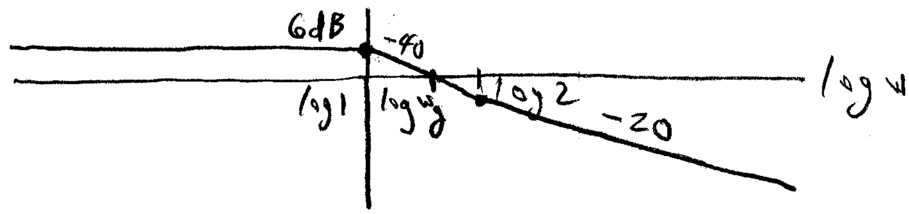
$= .33$

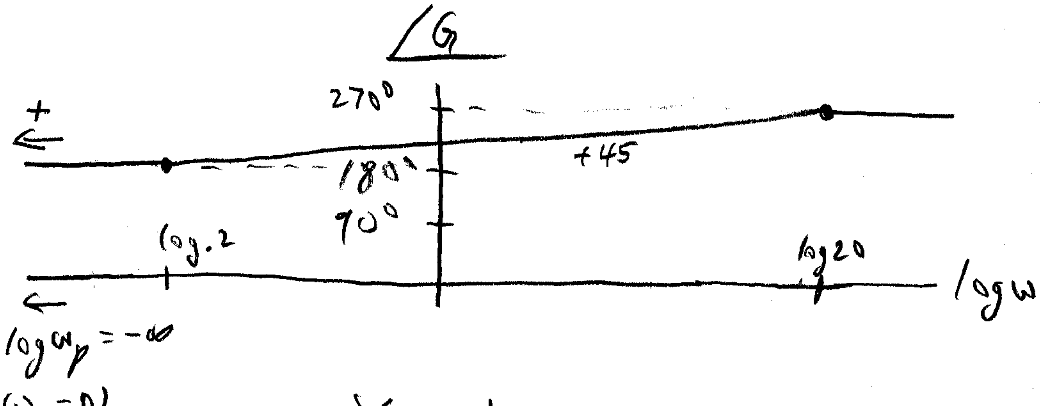
$\gamma = -2$

$\omega_p \neq \omega_g \Rightarrow G(\Gamma)$ does not pass through -1,

$p - \gamma = 2 \Rightarrow H$ has no imaginary pole and 2 open RHP poles.

b) $$G(s) = -2 \frac{\frac{s}{2}+1}{(\frac{s}{-1}+1)(s+1)} \qquad (p=1)$$

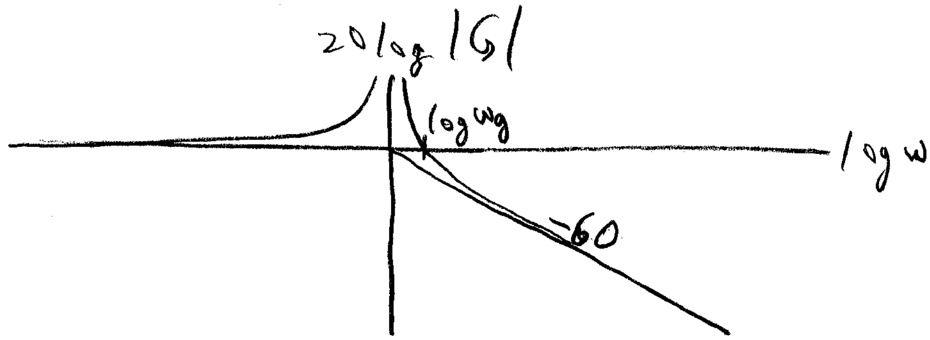


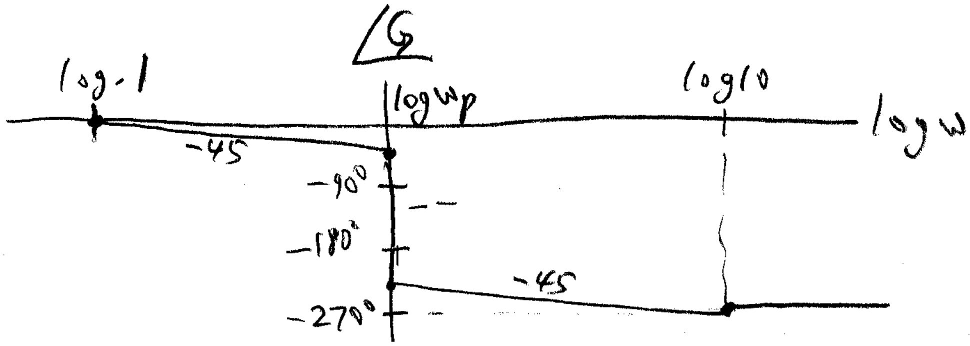


$\gamma = 1$

$p - \gamma = 0 \Rightarrow$ BIBO stable

c) $G(s) = \dfrac{1}{(s+1)(s^2+1)}$ $(p=0)$



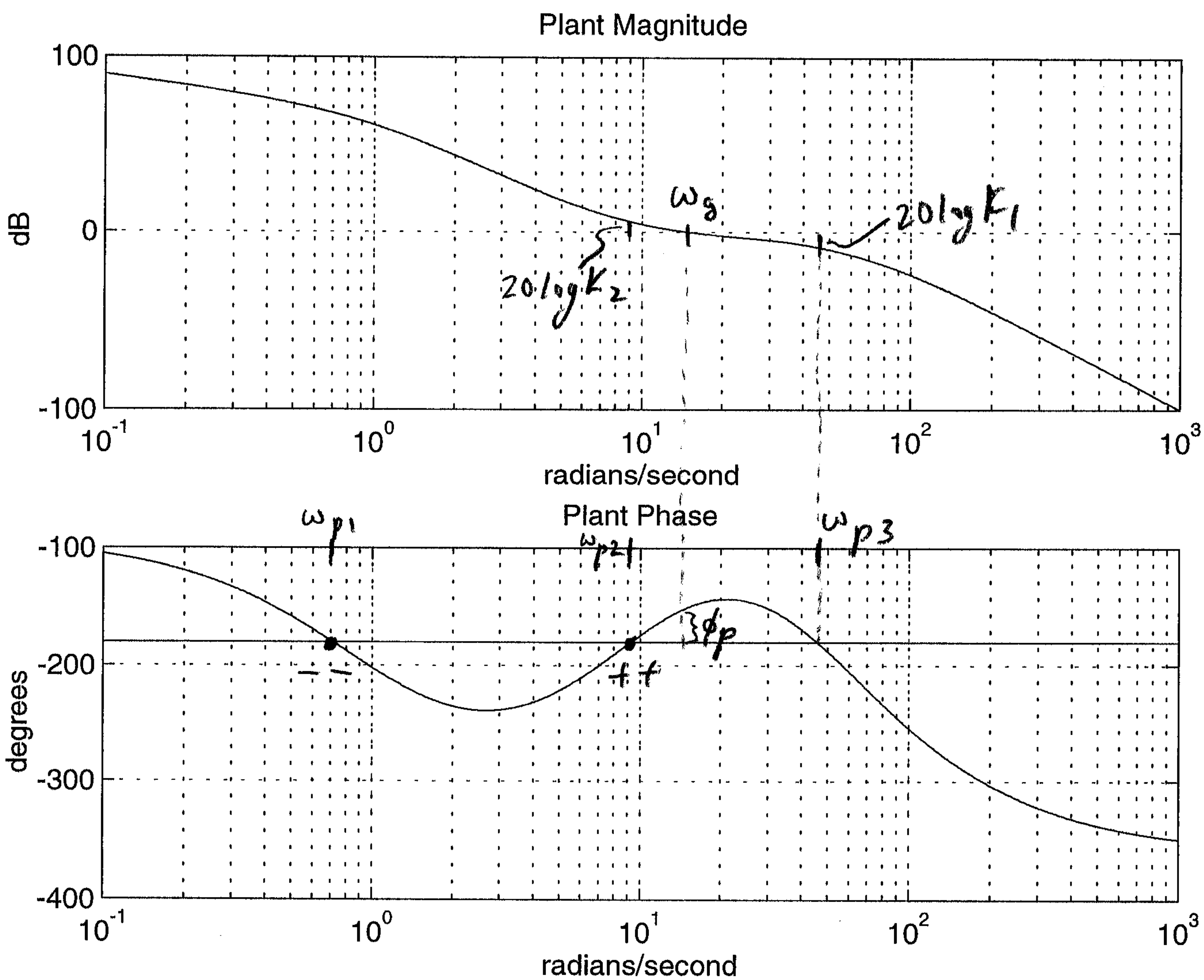


$\gamma = -2$

$\omega_p \neq \omega_g \Rightarrow G(\Gamma)$ does not pass through -1.

$p - \gamma = 2 \Rightarrow$ It has an imaginary pole and 2 open RHP poles.

4 a)



$\omega_g = 14$

$\omega_p = .7, 9.2, 46$

$\gamma_+ = 2, \gamma_- = 2, \gamma = 0, P = 0$

$P - \gamma = 0 \Rightarrow$ BIBO stable

b) G has 1 pole at 0, so $g = 1$.

$$e_{ss_1} = \lim_{s \to 0} \frac{1}{s\,G(s)} = \lim_{s \to 0} \frac{(s+1)^3 (s+50)^5}{10^7 (s+10)^5} = \frac{50^5}{10^{12}} = 3.12 \times 10^{-}$$

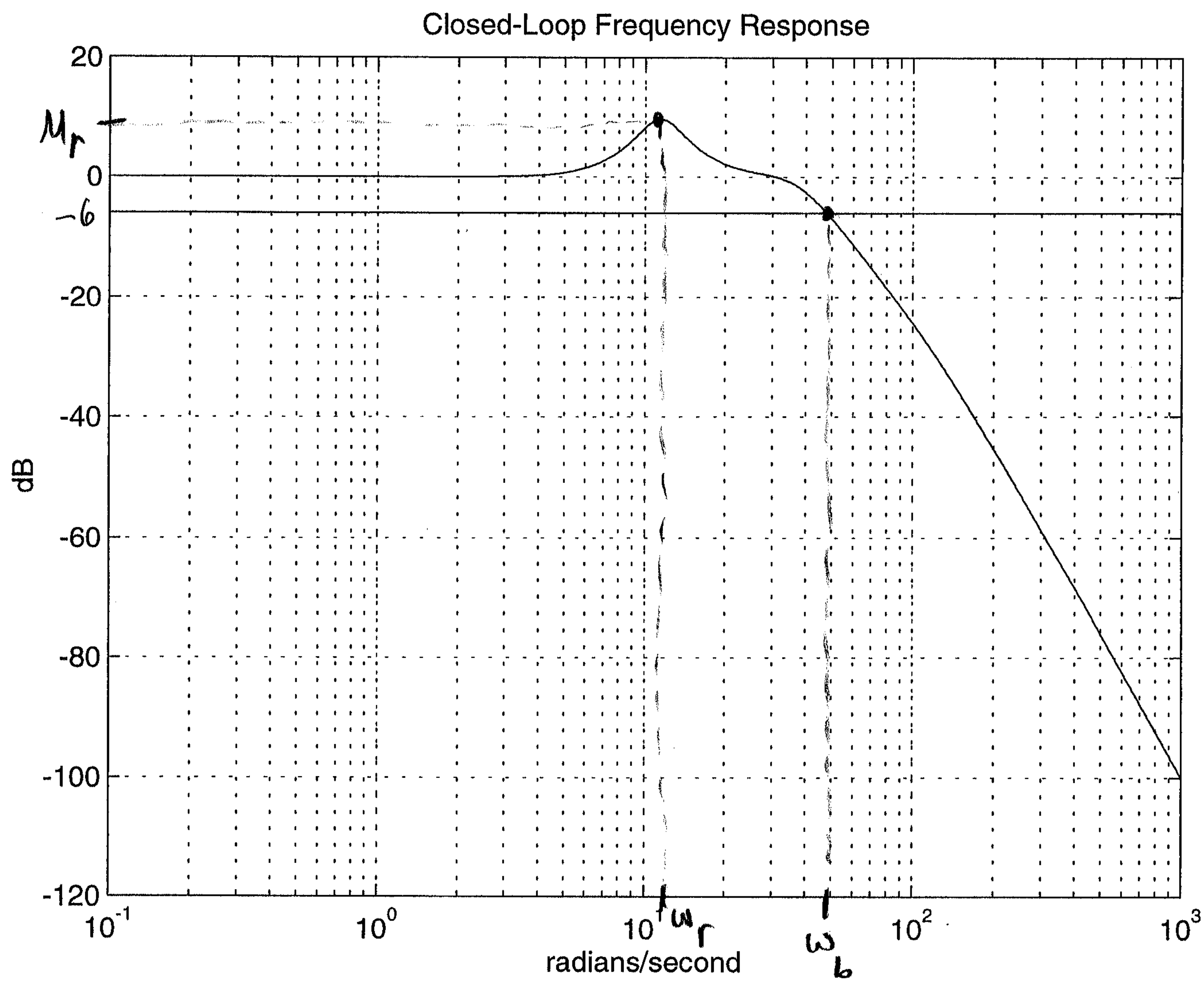
c) $\phi_p = 28^\circ$, $20 \log K_1 = 5\,dB$, $20 \log K_2 = 9\,dB$, $20 \log K_g = 5\,dB$

$\omega_b = 48 \frac{rad}{s}$, $\omega_r = 11 \frac{rad}{s}$, $M_r = 10\,dB$

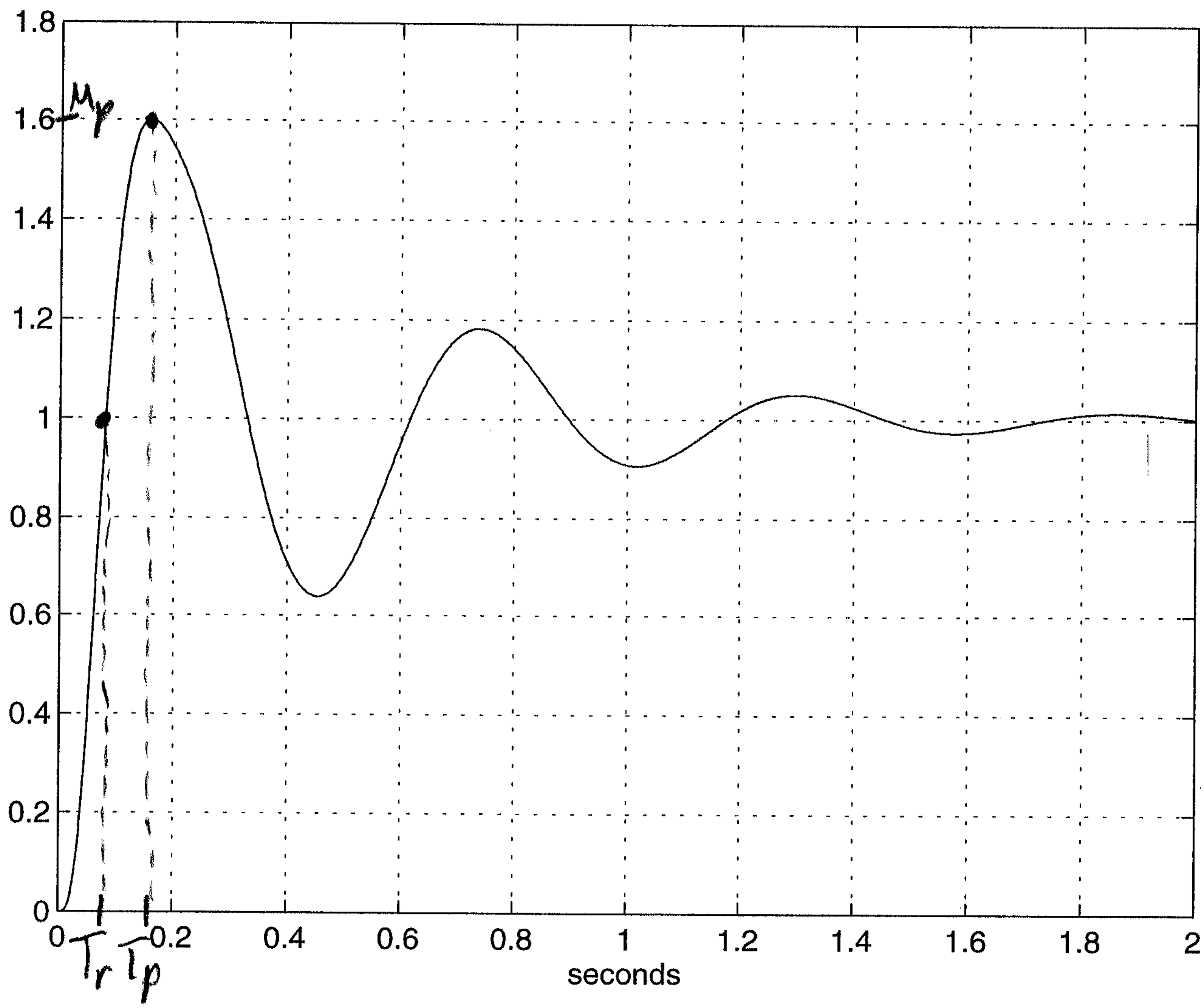
$T_r = .08\,s$, $T_p = .16\,s$, $M_p = 1.6$

d) $\omega_g = 14$, $\omega_b = 48$

$\omega_b \geq \omega_g$ (This must occur.)



Closed-Loop Frequency Response
20
0
-20
-40
-60
-80
-100
-120
dB
M_r
-6
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
radians/second
ω_r
ω_b



1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
seconds
M_p
T_r
T_p

5/ For a 2nd-order system,

$$\omega_g = \omega_n \sqrt{\sqrt{4\xi^4+1} - 2\xi^2},$$

$$\phi_p = 90^\circ - \tan^{-1} \frac{\sqrt{\sqrt{4\xi^4+1} - 2\xi^2}}{2\xi}.$$

For $\phi_p \geq 60^\circ$,

$$\tan^{-1} \frac{\sqrt{\sqrt{4\xi^4+1} - 2\xi^2}}{2\xi} \leq 30^\circ$$

$$\frac{\sqrt{\sqrt{4\xi^4+1} - 2\xi^2}}{2\xi} \leq \frac{1}{\sqrt{3}}$$

$$\sqrt{4\xi^4+1} - 2\xi^2 \leq \frac{4}{3}\xi^2$$

$$\xi^4 \geq \frac{9}{64}$$

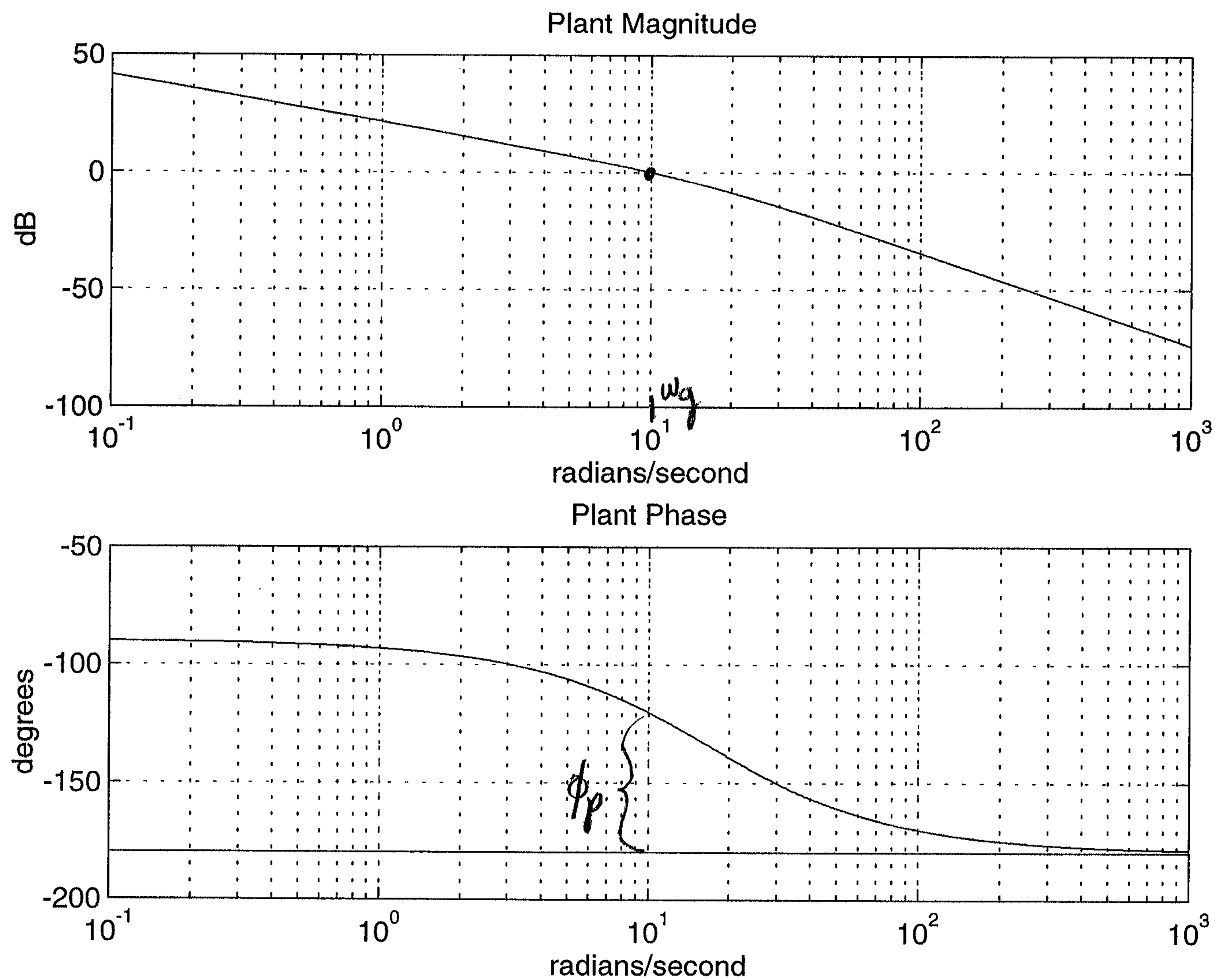
$$\xi \geq \frac{\sqrt{6}}{4} = .612$$

Take $\xi = .612$. Then we need

$$\omega_n \sqrt{\sqrt{4\xi^4+1} - 2\xi^2} \geq 10$$

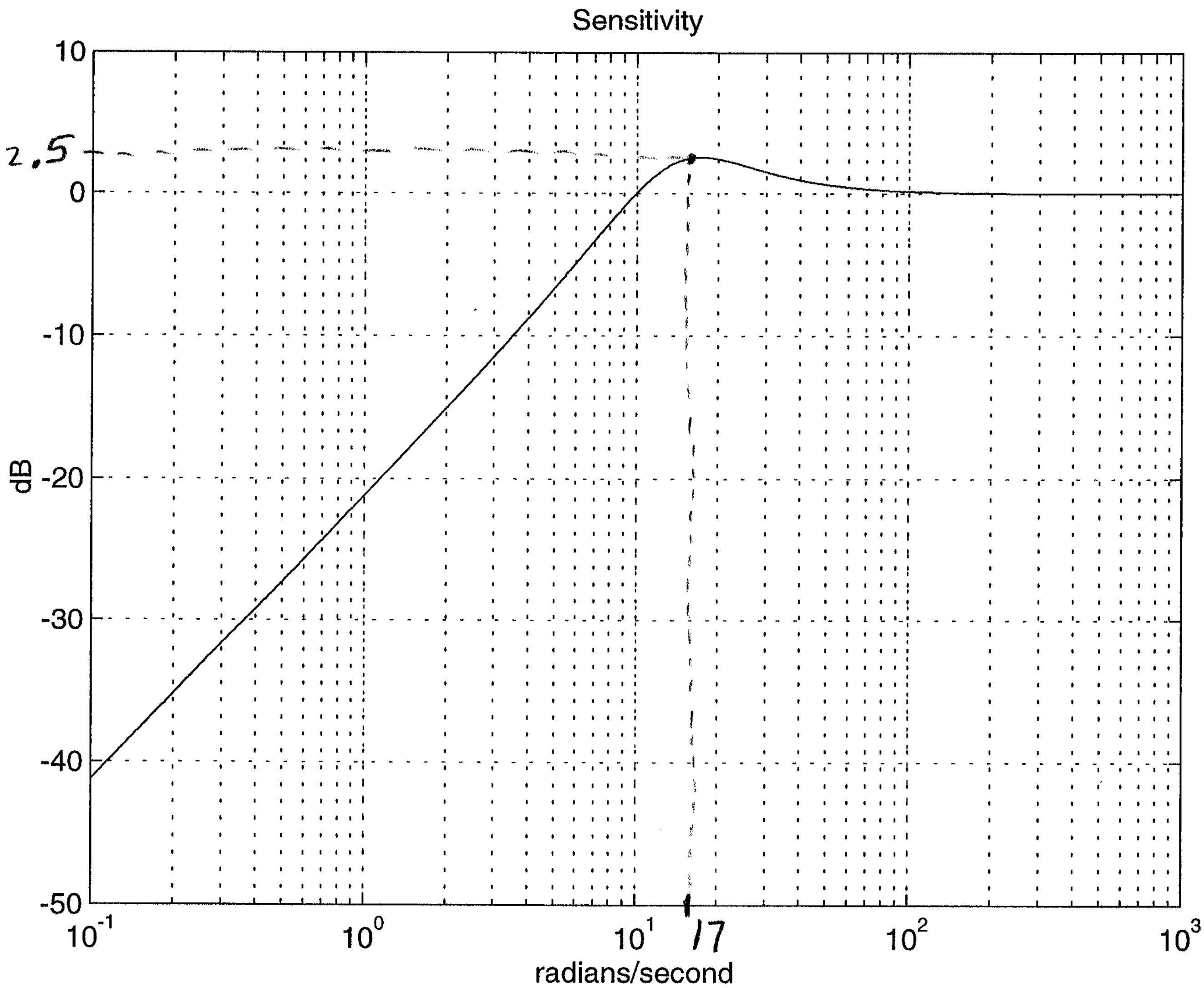
$$\omega_n \geq \frac{10}{\sqrt{\sqrt{4\xi^4+1} - 2\xi^2}} = 14.1$$

Set $\omega_n = 14.1 \ \frac{rad}{s}$.



Plant Magnitude
50
0
-50
-100
dB
10^-1
10^0
10^1
10^2
10^3
radians/second
ωg
Plant Phase
-50
-100
-150
-200
degrees
10^-1
10^0
10^1
10^2
10^3
radians/second
φp

6)


Sensitivity
10
2.5
0
-10
-20
-30
-40
-50
dB
10^-1
10^0
10^1
17
10^2
10^3
radians/second