# ECE 332
# Homework #6

1) Consider the servomotor

$$G(s) = \frac{1}{s^3 + s^2 + s}.$$

Design a lead-lag compensator to achieve the specifications

a) $e_{ss1} \leq .5$
b) $\omega_g' \geq 1$
c) $\phi_p' \geq 60°$

Two MATLAB routines will help in this design. First, fnd the gain constant $K$ and the lead and lag poles and zeros. The final Bode plots for the compensator and loop transfer function, the closed-loop frequency response, and the closed-loop step response can be generated by typing "$k =$ _; $a =$ _; $b =$ _; $c =$ _; $d =$ _; $hw61$". Read the values of $\omega_b$, $\omega_r$, $M_r$, $T_r$, $T_p$, and $M_p$ from the graphs.

2) Repeat problem 1) for the specifications

a) $e_{ss1} \leq .01$
b) $\omega_g' \geq 4$
c) $\phi_p' \geq 75°$

The appropriate MATLAB command is "$k =$ _; $a =$ _; $b =$ _; $c =$ _; $d =$ _; $stages =$ _; $hw62$", where "$stages$" is the number of stages in your compensator. Compare your results to those obtained in problem 1).

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Homework #6
Solutions

1) $e_{ss1} = \lim_{s \to 0} \frac{1}{sKG(s)} = \frac{1}{K} \leq .5$

Let $K = 2$, $\omega_g' = 1$.

$K|G(j\omega_g')| = \left|\frac{2}{-j-1+j}\right| = 2$, $\angle G(j\omega_g') = \angle \frac{1}{-j-1+j} = -180^\circ$

$\phi_0 = \phi_p' - \angle G(j\omega_g') - 180^\circ + 6^\circ$

$= 60 + 180 - 180 + 6$

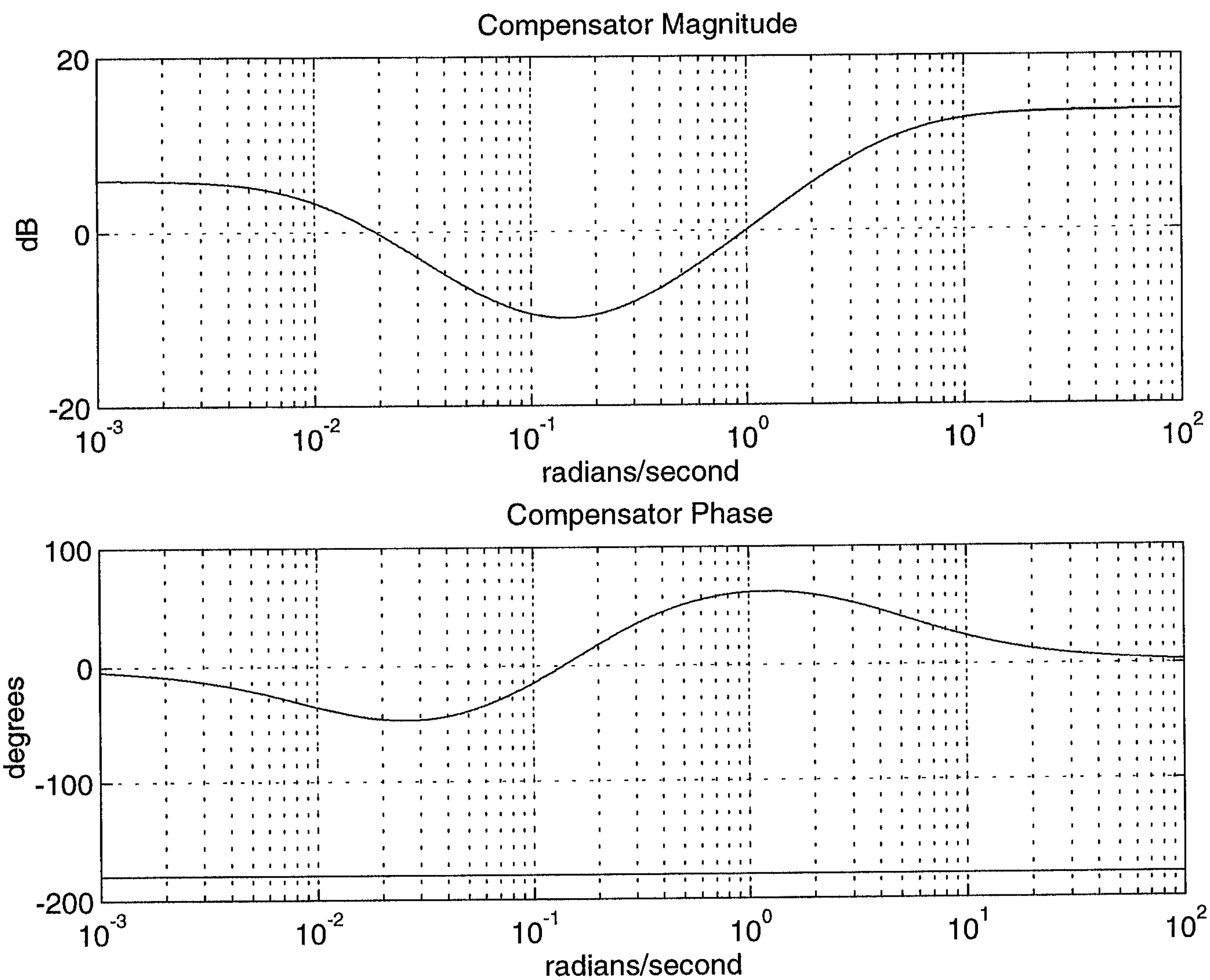
$= 66^\circ$

$d = \omega_g' \tan\left(\frac{\phi_0 + 90}{2}\right) = \tan\left(\frac{66+90}{2}\right) = 4.7$

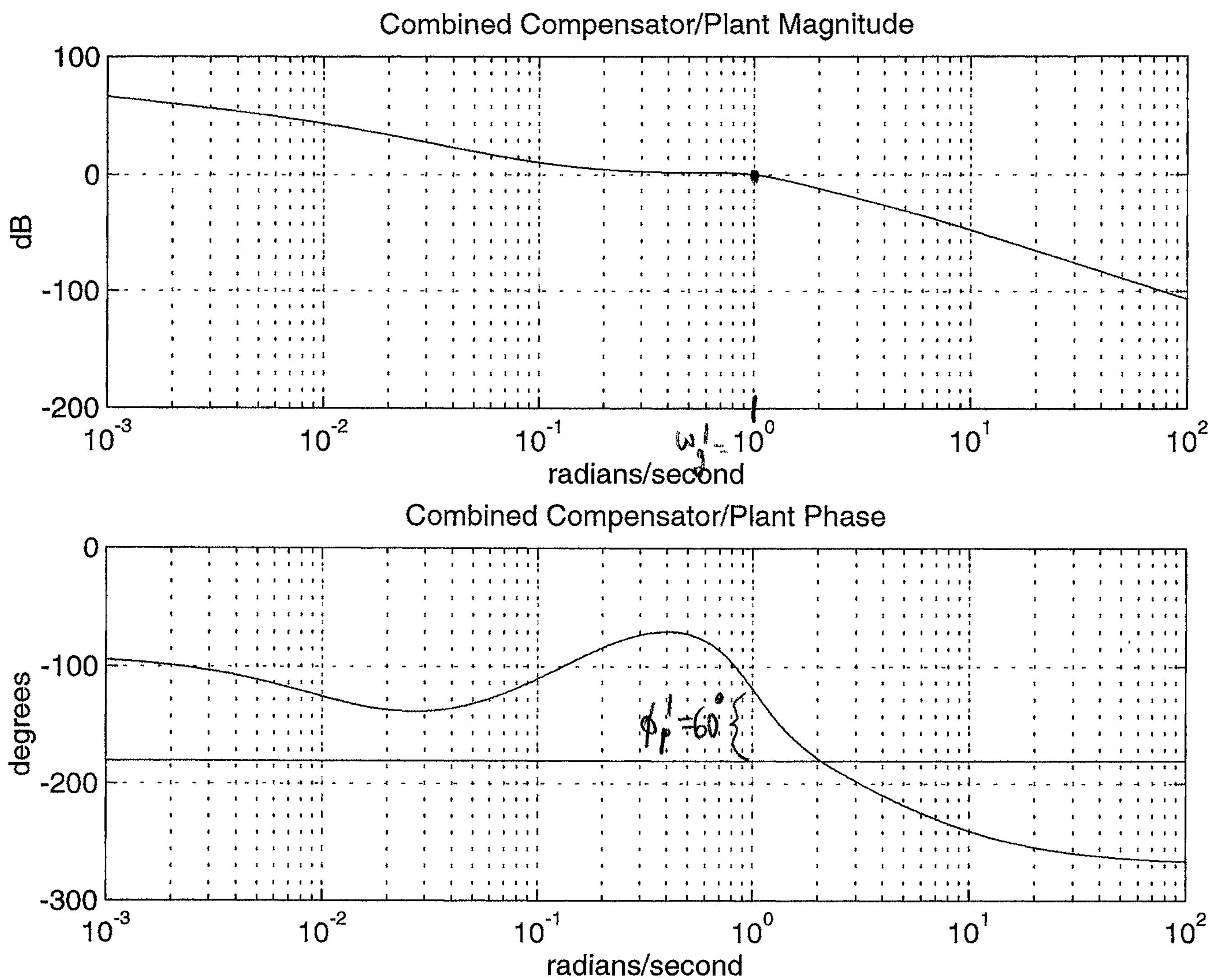
$c = \frac{(\omega_g')^2}{d} = \frac{1}{4.7} = .21$

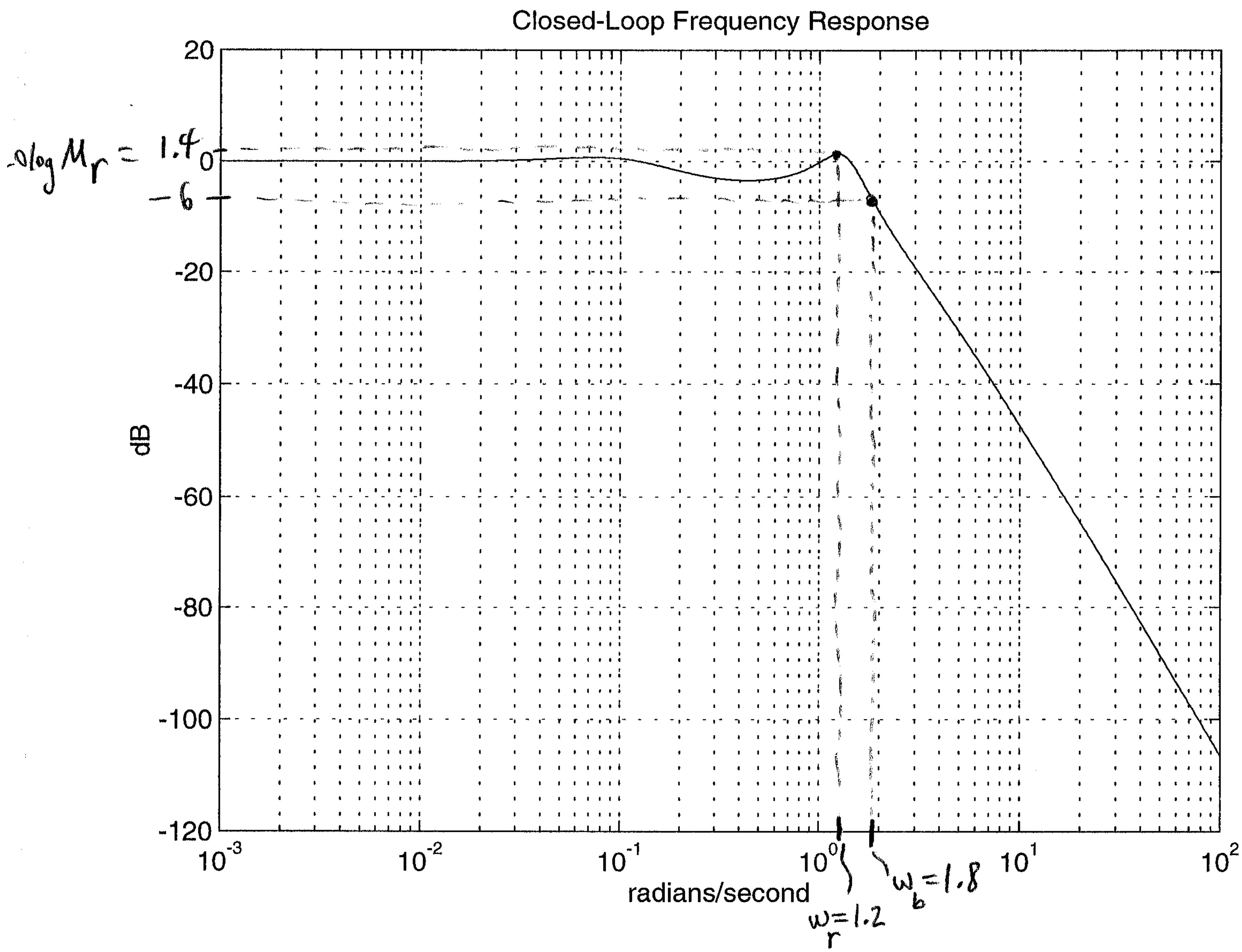
$b = \frac{\omega_g'}{10} = .1$

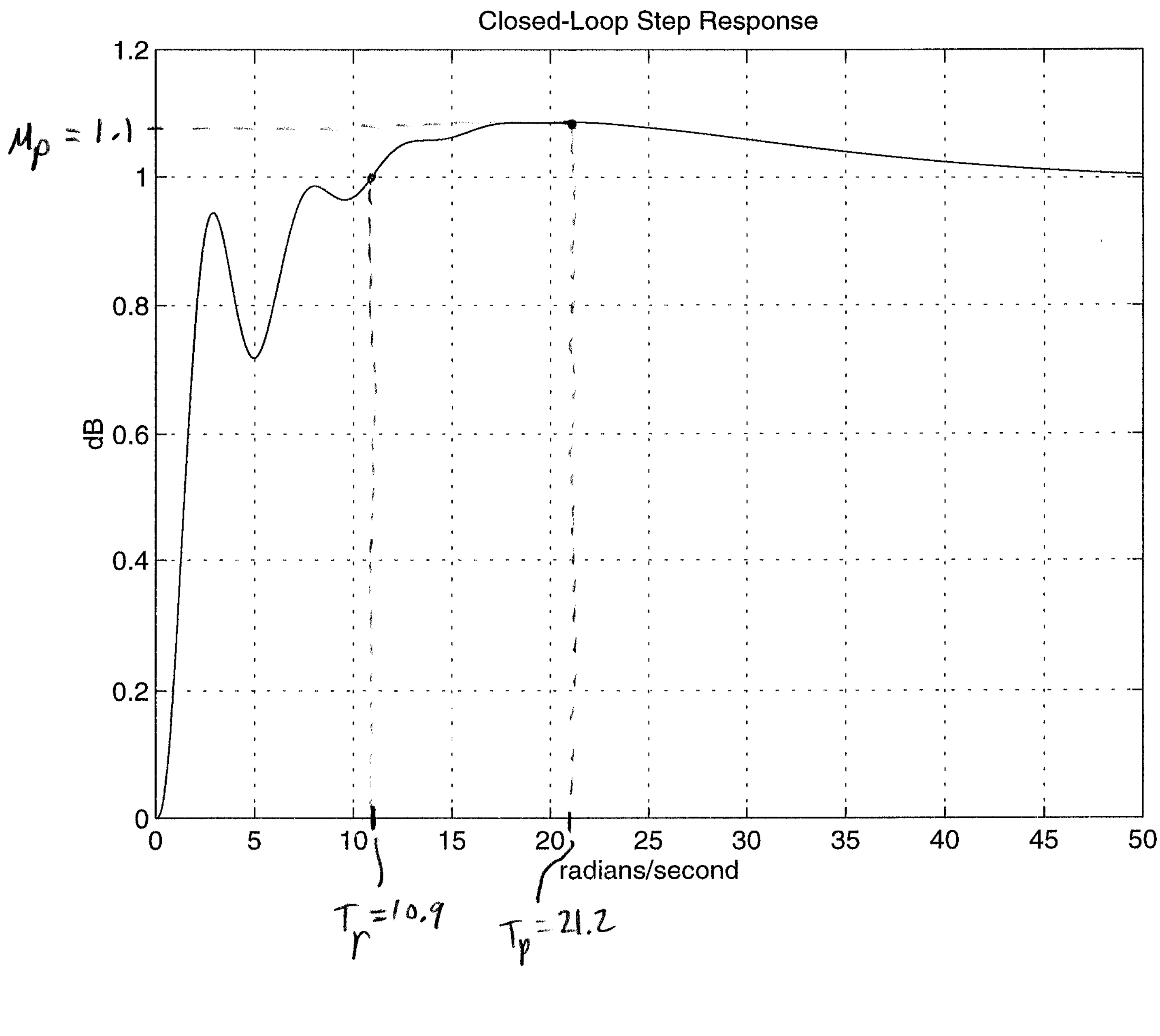
$a = \frac{b}{K|G(j\omega_g')|\sqrt{\frac{d}{c}}} = \frac{.1}{2\sqrt{\frac{4.7}{.21}}} = .01$



Compensator Magnitude
20
0
-20
dB
10^-3
10^-2
10^-1
10^0
10^1
10^2
radians/second
Compensator Phase
100
0
-100
-200
degrees
10^-3
10^-2
10^-1
10^0
10^1
10^2
radians/second


Combined Compensator/Plant Magnitude
dB
100
0
-100
-200
10^-3
10^-2
10^-1
10^0
10^1
10^2
ωg' =
radians/second
Combined Compensator/Plant Phase
degrees
0
-100
-200
-300
φp' = 60°
10^-3
10^-2
10^-1
10^0
10^1
10^2
radians/second


Closed-Loop Frequency Response
20
0
-20
-40
-60
-80
-100
-120
dB
$10^{-3}$
$10^{-2}$
$10^{-1}$
$10^{0}$
$10^{1}$
$10^{2}$
radians/second
$20 \log M_r = 1.4$
$-6$
$\omega_r = 1.2$
$\omega_b = 1.8$


Closed-Loop Step Response
dB
radians/second
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30
35
40
45
50
$M_p = 1.1$
$T_r = 10.9$
$T_p = 21.2$

2) $e_{ss1} = \frac{1}{K} \leq .01$

Let $K = 100$, $\omega_g' = 4$.

$$K|G(j\omega_g')| = \left|\frac{100}{-j64 - 16 + j4}\right| = \left|\frac{100}{-16 - j60}\right| = \frac{100}{\sqrt{16^2 + 60^2}} = 1.61$$

$$\angle G(j\omega_g') = \angle \frac{1}{-16 - j60} = -180^\circ - \tan^{-1}\frac{60}{16} = -255^\circ$$

For a single lead stage,

$$\phi_\ell = \phi_r' - \angle G(j\omega_g') - 180^\circ + 6^\circ$$

$$= 75 + 255 - 180 + 6$$

$$= 156^\circ$$

Two leads are required, each supplying

$$\phi_\ell = \frac{\phi_r' - \angle G(j\omega_g') - 180^\circ}{n} + 6^\circ$$

$$= \frac{75 + 255 - 180}{2} + 6$$

$$= 81^\circ$$

Chose lead.

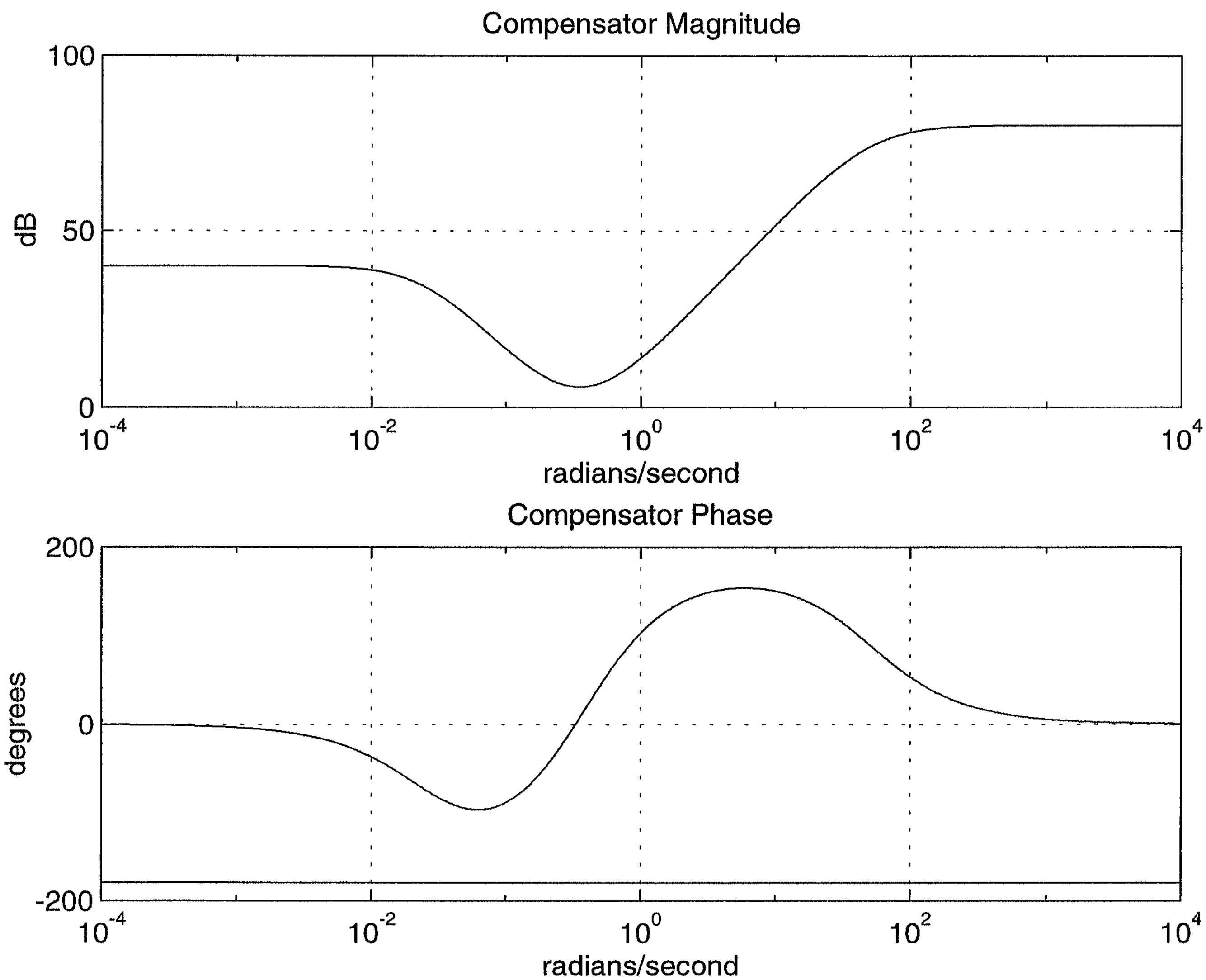
$$d = \omega_g' \tan\left(\frac{\phi_e + 90}{2}\right) = 4 \tan\left(\frac{81+90}{2}\right) = 51.0$$

$$c = \frac{(\omega_g')^2}{d} = \frac{4^2}{63.8} = .31$$

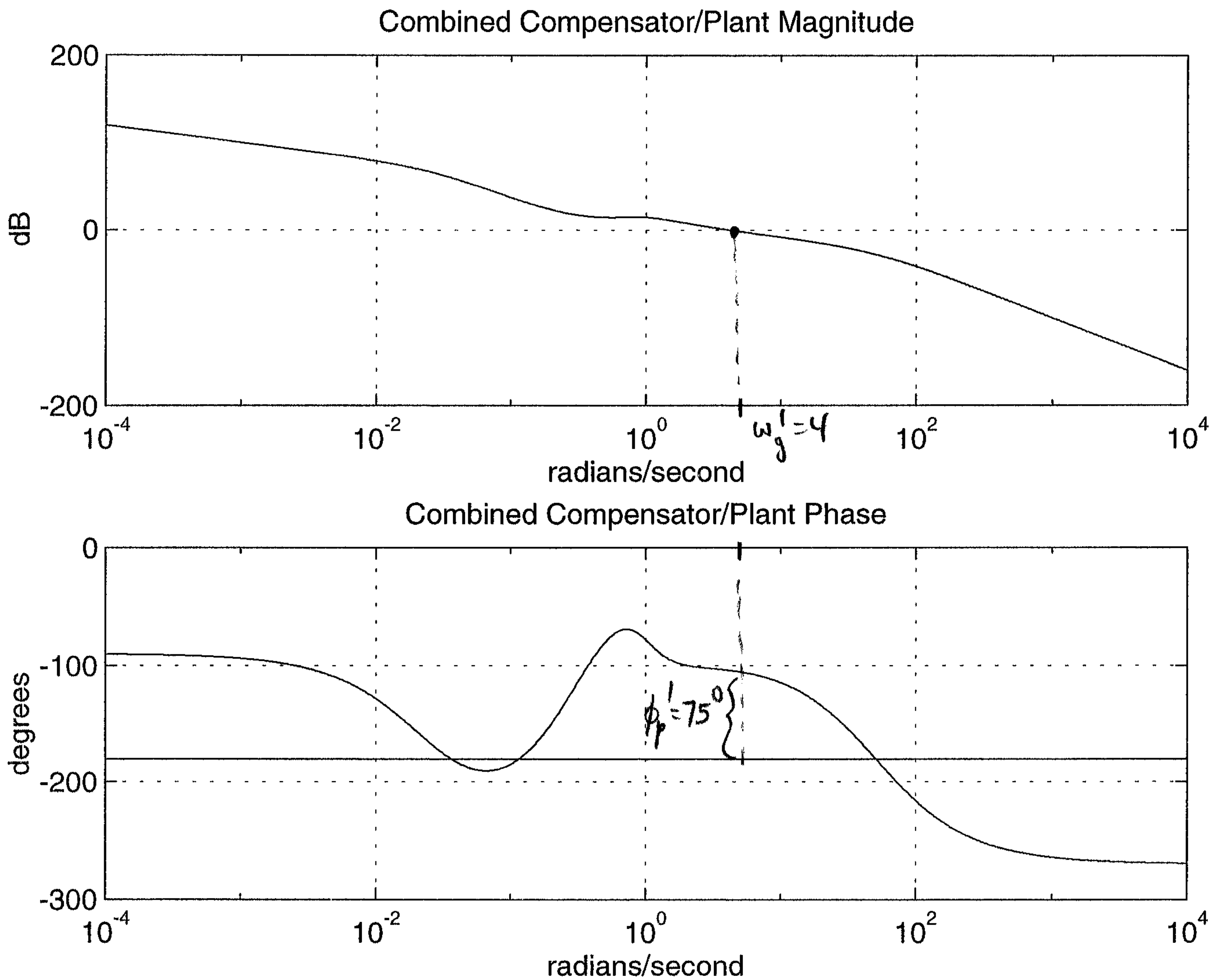
$$b = \frac{\omega_g'}{10} = \frac{4}{10} = .4$$

$$a = \frac{b}{\sqrt{K\,|G(j\omega_g')|}\sqrt{\frac{d}{c}}} = \frac{.4}{\sqrt{1.61}\sqrt{\frac{63.8}{.25}}} = .02$$

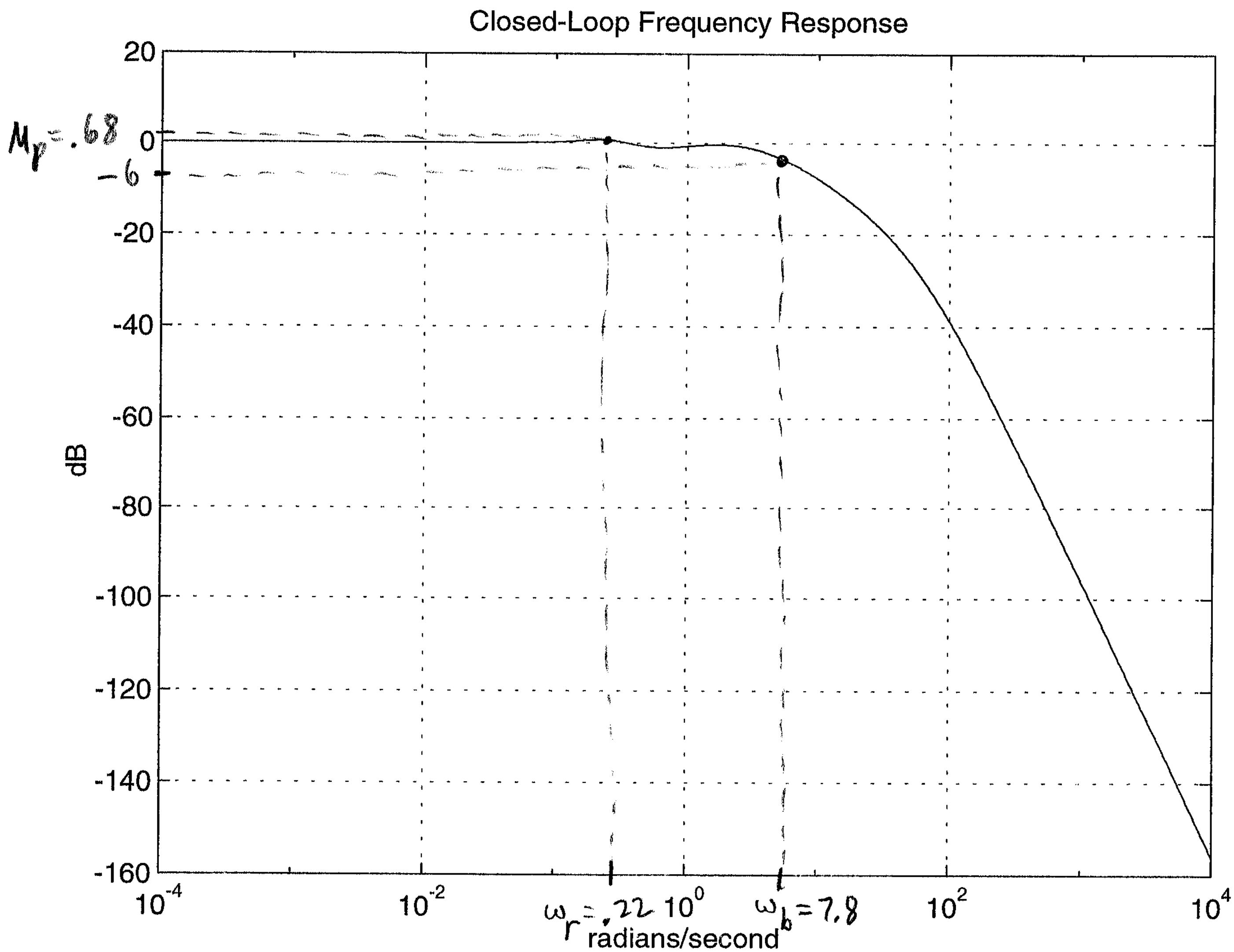
Two lead and lag stages yield larger bandwidth and smaller rise time with a slight decrease in peaking in both the frequency and step responses.



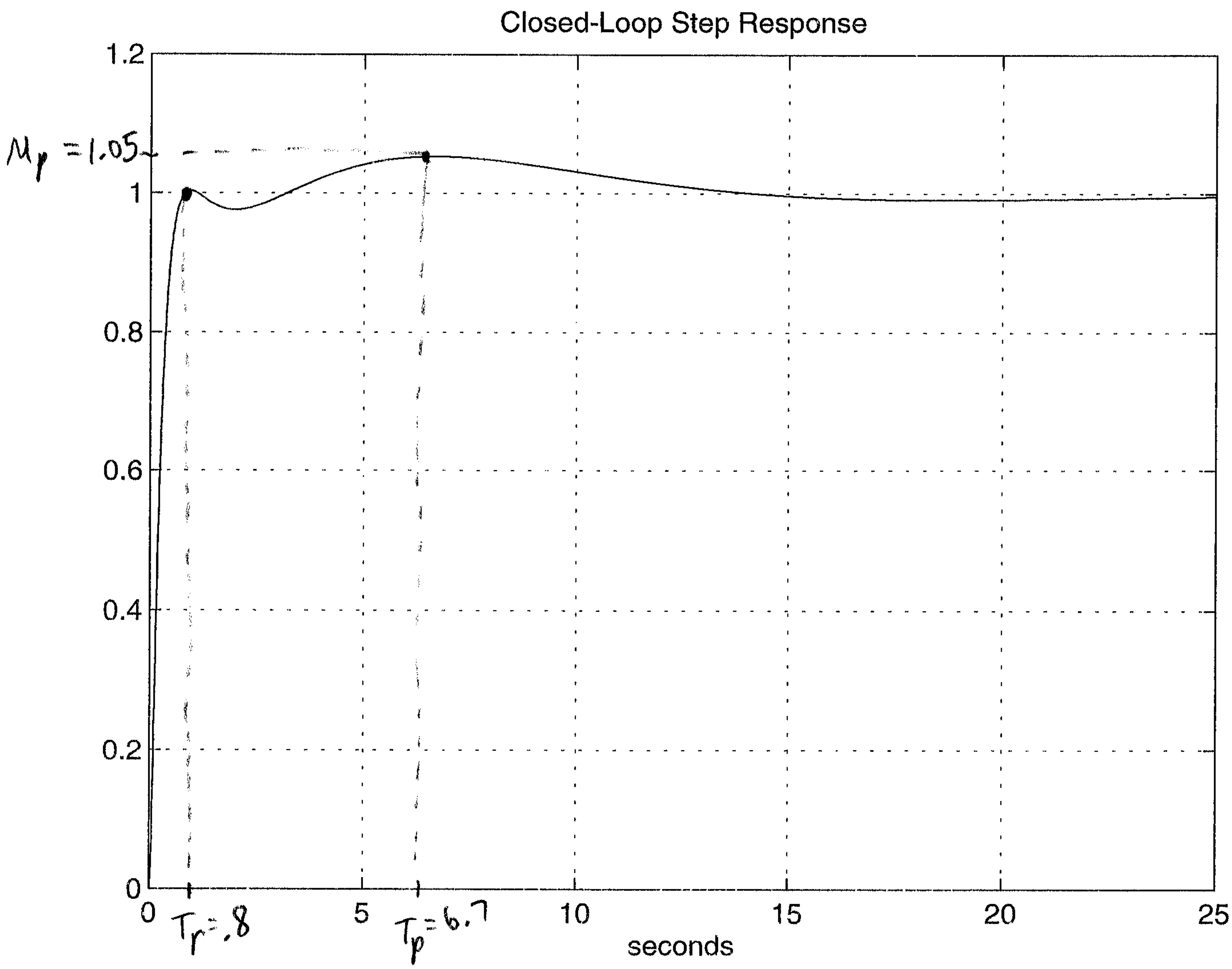
Compensator Magnitude
100
50
0
dB
10^-4
10^-2
10^0
10^2
10^4
radians/second
Compensator Phase
200
0
-200
degrees
10^-4
10^-2
10^0
10^2
10^4
radians/second



Combined Compensator/Plant Magnitude
200
0
-200
dB
10^-4
10^-2
10^0
10^2
10^4
radians/second
ω'g = 4
Combined Compensator/Plant Phase
0
-100
-200
-300
degrees
φ'p = 75°
10^-4
10^-2
10^0
10^2
10^4
radians/second



Closed-Loop Frequency Response
20
0
-20
-40
-60
-80
-100
-120
-140
-160
dB
$M_r = .68$
$-6$
$10^{-4}$
$10^{-2}$
$10^{0}$
$10^{2}$
$10^{4}$
$\omega_r = .22$
$\omega_b = 7.8$
radians/second


Closed-Loop Step Response
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
seconds
$M_p = 1.05$
$T_r = .8$
$T_p = 6.7$