ECE 332
Homework #7

1) Consider the plant

$$G(s) = \frac{s+2}{(s+1)^3}.$$

Using algebraic methods, design a proper compensator so that the closed-loop system is asymptotically stable and satisfies the specifications

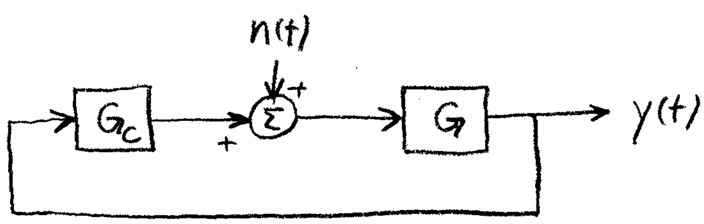
a) $e_{ss0} = 0$
b) $M_r \le 1.3$
c) $M_p \le 1.3$
d) $e_{ss1} \le .8\ s$
e) $\omega_b \ge 1.5\ rad/s$
f) $\omega_r \ge .7\ rad/s$
g) $T_r \le 2.3\ s$
h) $T_p \le 3.5\ s.$

2) Consider the plant

$$G(s) = \frac{(s-2)^2}{s(s-1)(s^2+.01s+1)}.$$

a) Design a compensator to make the closed-loop system asymptotically stable with all poles at $s = -1$. In MATLAB, display the closed-loop impulse response corresponding to your compensator by typing "nc = _; dc = _; $hw72i$", filling in the blanks with the numerator and denominator of your compensator. (In MATLAB, a polynomial is represented as a row vector. For example, to enter $p(s) = as^2 + bs + c$, type "[a b c]".)

b) Consider the closed-loop system with noise input as shown.



In MATLAB, type "nc = _; dc = _; $hw72ii$" to plot the impulse response from $n(t)$ to $y(t)$, using your compensator from part a). Explain the result.

3) Repeat problem 2) under the additional constraint that all closed-loop eigenvalues have damping angle $\psi \ge 45°$. Use the same MATLAB routines as in problem 2) and explain your observations.

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Solutions

1) G is stable, minimum phase, so any BIBO stable H with

$$\text{rel } H \geq \text{rel } G = 3-1 = 2$$

is achievable. Hence, we are free to choose a 2nd-order transfer function

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}.$$

From Homework #3, problem 1), the given specifications are achieved by

$$\xi = .43, \quad \omega_n = 1.08.$$

$$H(s) = \frac{1.17}{s^2 + .93s + 1.17}$$

$$G_c(s) = \frac{N_{CL} D}{N(D_{CL} - N_{CL})} = \frac{1.17(s+1)^3}{s(s+2)(s+.93)}$$

2a) The plant is unstable, non-minimum phase.

1) $N^+ = (s-2)^2$, $N^- = 1$, $D^+ = s(s-1)$, $D^- = s^2 + .01s + 1$

2) $\deg D_{CL} \geq \deg N^+ + \deg D^+ + \text{rel}\, G - 1 = 2+2+2-1 = 5$

$$D_{CL} = (s+1)^5$$

3) Solve $N^+ X + D^+ Y = 1$.

Divide D^+ by N^+:

$$\begin{array}{rr|rrr} 4 & -4 & 1 & -1 & 0 \\ & & & 4 & -4 \\ \hline & & 1 & 3 & 4 \end{array}$$

$Q_1 = 1$

$R_1 = 3s - 4$

Divide N^+ by $R_1 = 3(s - \frac{4}{3})$

$$\begin{array}{r|rrr} \frac{4}{3} & 1 & -4 & 4 \\ & & \frac{4}{3} & -\frac{32}{9} \\ \hline & 1 & -\frac{8}{3} & \frac{4}{9} \end{array}$$

$Q_2 = \frac{1}{3}s - \frac{8}{9}$

$R_2 = \frac{4}{9}$

$X_1 = -Q_1 = -1, \quad Y_1 = 1$

$X_2 = 1 - Q_2 X_1 = \frac{1}{3}s + \frac{1}{9}, \quad Y_2 = -Q_2 Y_1 = -\frac{1}{3}s + \frac{8}{9}$

$X = \frac{X_2}{R_2} = \frac{3}{4}s + \frac{1}{4}, \quad Y = \frac{Y_2}{R_2} = -\frac{3}{4}s + 2$

Check: $(\frac{3}{4}s + \frac{1}{4})(s^2 - 4s + 4) + (-\frac{3}{4}s + 2)(s^2 - s) = 1$

4) $X D_{CL} = (\frac{3}{4}s + \frac{1}{4})(s+1)^5$

$= \frac{1}{4}(3s^6 + 16s^5 + 35s^4 + 40s^3 + 25s^2 + 8s + 1)$

Divide by $-D^+ = -(s^2 - s)$:

$$\begin{array}{cc|ccccccc}
1 & 0 & 3 & 16 & 35 & 40 & 25 & 8 & 1 \\
 & & & & 0 & 0 & 0 & 0 & 0 \\
 & & & 3 & 19 & 54 & 94 & 119 & \\
\hline
 & & 3 & 19 & 54 & 94 & 119 & \boxed{127} & \boxed{1}
\end{array}$$

$M = -\frac{1}{4}(3s^4 + 19s^3 + 54s^2 + 94s + 119)$

$\tilde{N} = \frac{1}{4}(127s + 1)$

5)

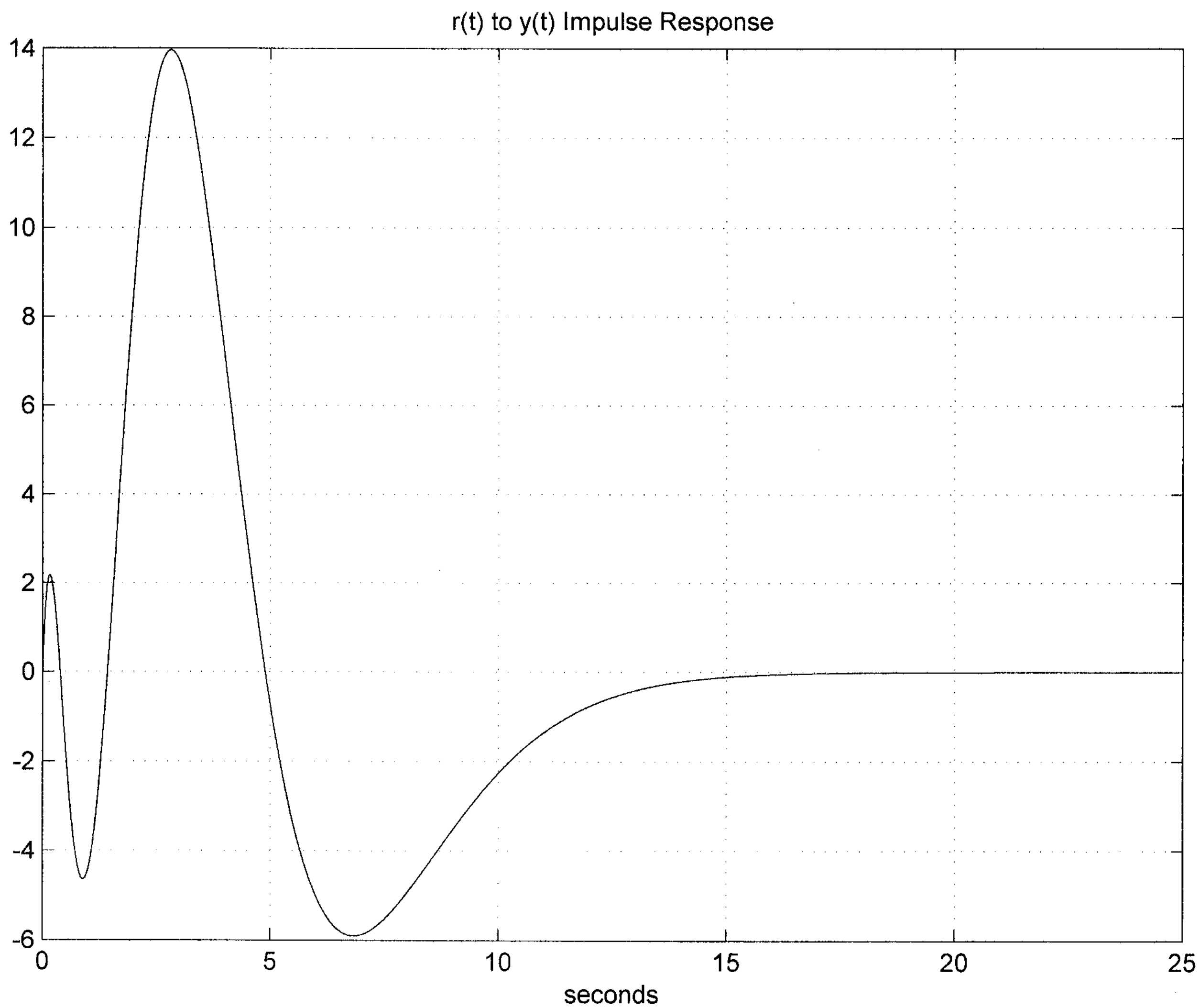
$$Y D_{CL} - N^{+} M = \left(-\tfrac{3}{4}s+2\right)(s+1)^5 - (s-2)^2 M$$

$$= s^3 + 6s^2 - \tfrac{63}{4}s + 121$$

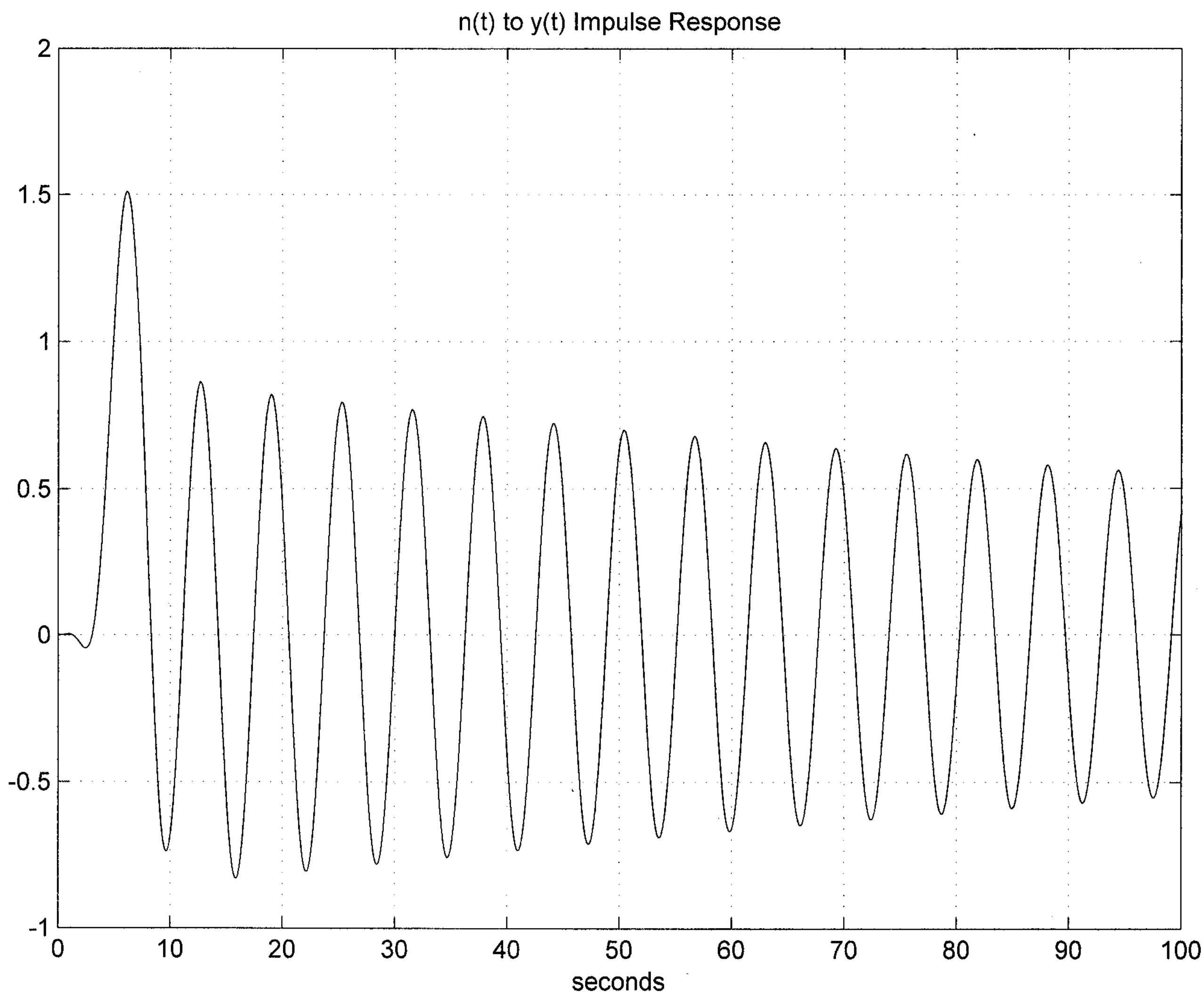
$$G_c = \frac{D^- \tilde{N}}{N^-(Y D_{CL} - N^+ M)}$$

$$= \frac{\tfrac{1}{4}(s^2 + .01s + 1)(.127s + 1)}{s^3 + 6s^2 - \tfrac{63}{4}s + 121}$$

$$H = \frac{N^+ \tilde{N}}{D_{CL}} = \frac{\tfrac{1}{4}(s-2)^2(.127s+1)}{(s+1)^5}$$


r(t) to y(t) Impulse Response
14
12
10
8
6
4
2
0
-2
-4
-6
0
5
10
15
20
25
seconds

b)


n(t) to y(t) Impulse Response
2
1.5
1
0.5
0
-0.5
-1
0
10
20
30
40
50
60
70
80
90
100
seconds

The graph shows the impulse response of

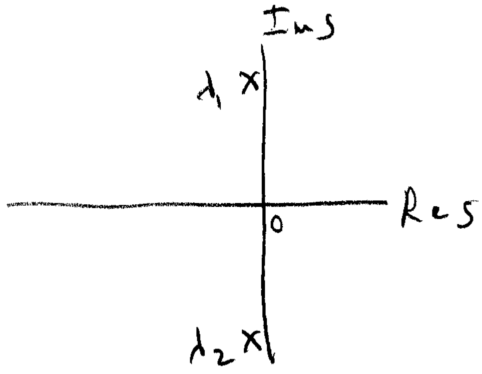
$$H_u = \frac{G}{1+G_c G} = \frac{H}{G_c} = \frac{N^+(YD_{CL}-N^+M)}{D^- D_{CL}}.$$

But

$$D^- = s^2 + .01s + 1,$$

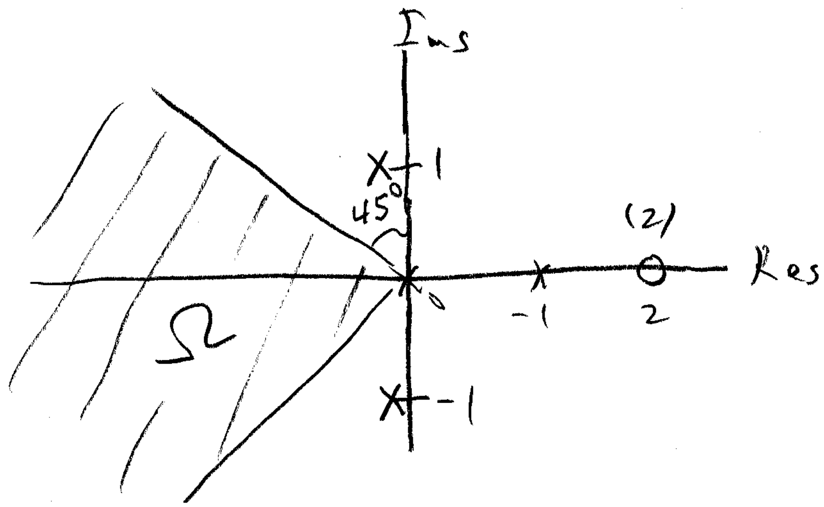
$$YD_{CL} - N^+M = s^3 + 6s^2 - \tfrac{63}{4}s + 121$$

are coprime, so H_u inherits the plant poles $\lambda_{1,2} = -.005 \pm j1.00$.



Hence, the impulse response oscillates with very little damping. While technically stable, the hidden modes may not be acceptable. These can be eliminated by imposing a damping angle.

3)



1) In this case, the complex poles do not lie in Ω, so they must be included in D^+.

$$N^+ = (s-2)^2, \quad N^- = 1, \quad D^+ = s(s-1)(s^2+.01s+1), \quad D^- = 1$$

2) $\deg D_{CL} \geq 2+4+2-1 = 7$

$$D_{CL} = (s+1)^7$$

3) $X = .229s^3 + .132s^2 + .389s + .25$

$Y = -.229s + .558$

4,5) $M = -.229s^6 - 1.96s^5 - 7.83s^4 - 19.8s^3 - 36.4s^2 - 52.5s - 62.5$

$\tilde{N} = 64.4s^3 - 1.30s^2 + 64.7s + .25$

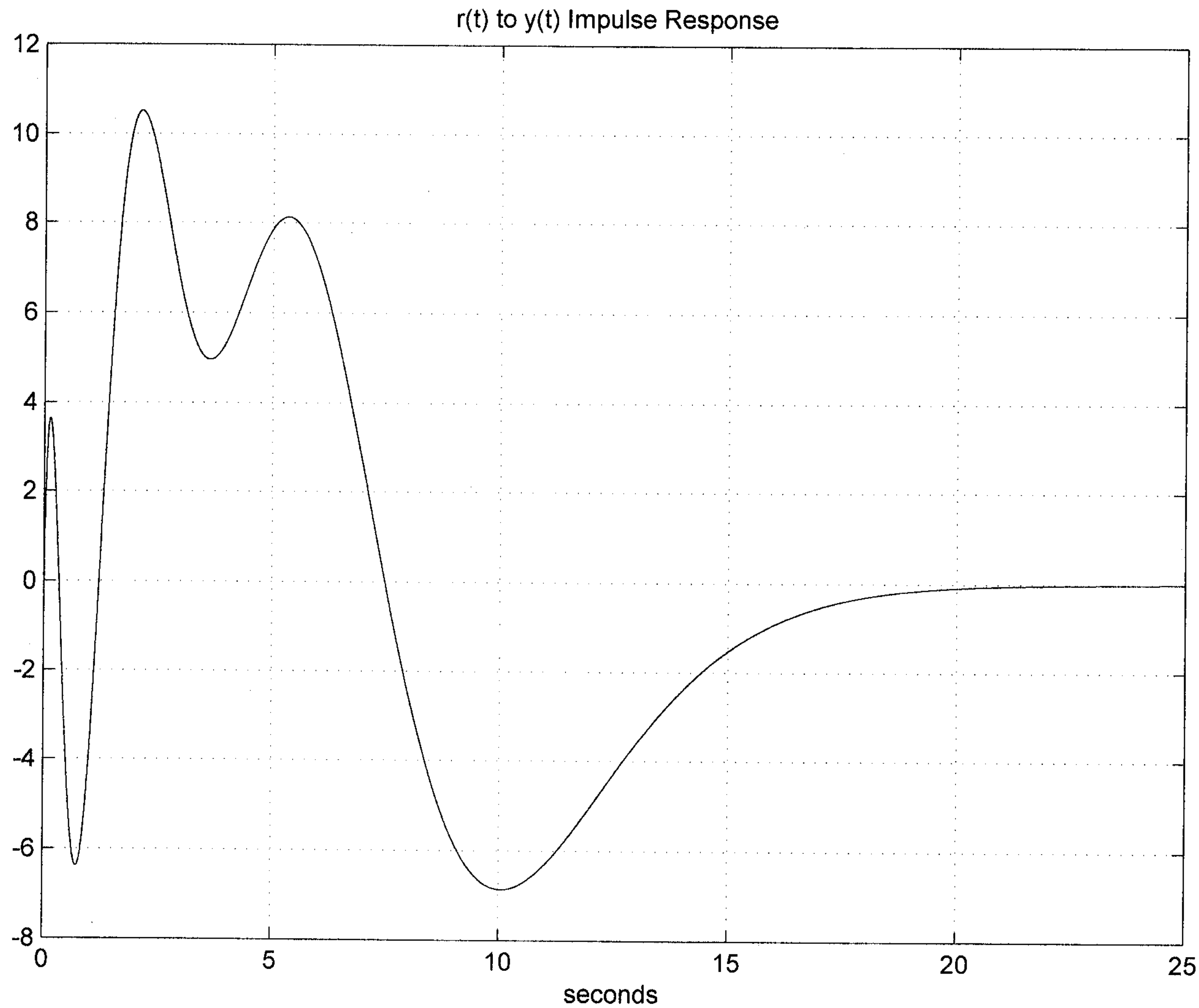
$$G_c = \frac{64.4s^3 - 1.30s^2 + 64.7s^2 + .25}{s^3 + 8.00s^2 - 36.4s + 251}$$

$$H = \frac{(s-2)^2(64.4s^3 - 1.30s^2 + 64.7s^2 + .25)}{(s+1)^7}$$

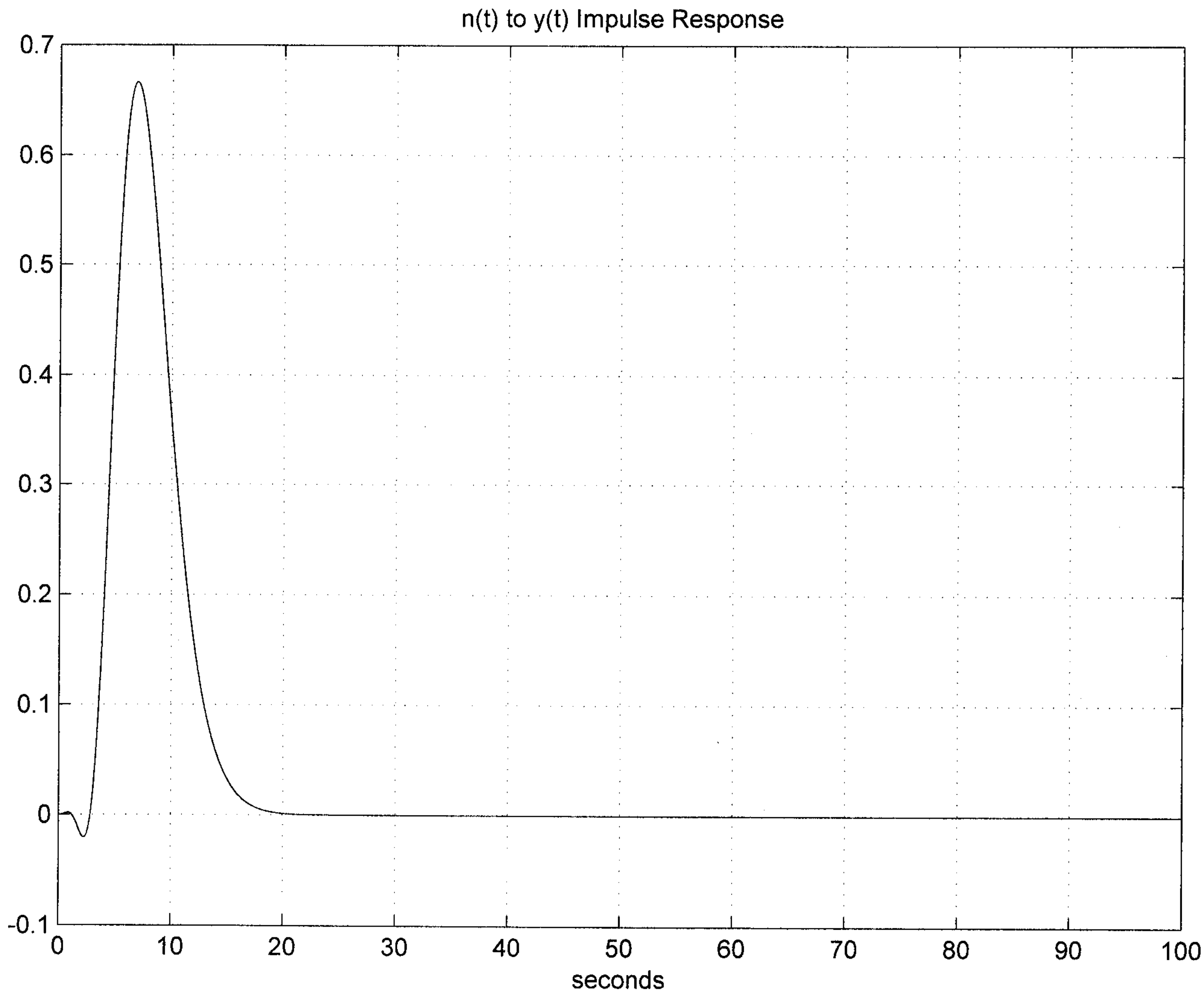
Now we obtain

$$H_u = \frac{N^+(YD_{CL} - N^+M)}{D_{CL}}.$$

Since the poorly damped plant poles are no longer poles of H_u, the impulse response from $u \to y$ is well-behaved.



r(t) to y(t) Impulse Response
12
10
8
6
4
2
0
-2
-4
-6
-8
0
5
10
15
20
25
seconds


n(t) to y(t) Impulse Response
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
10
20
30
40
50
60
70
80
90
100
seconds