ECE 332
Spring 2008
Final Exam

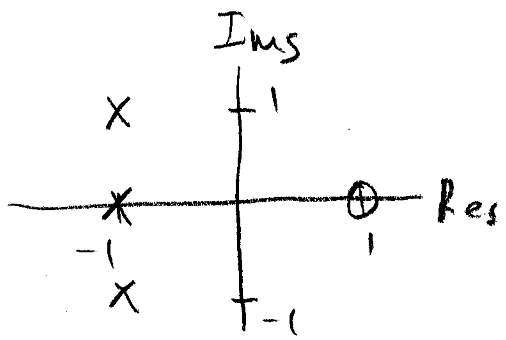
Name: Solutions

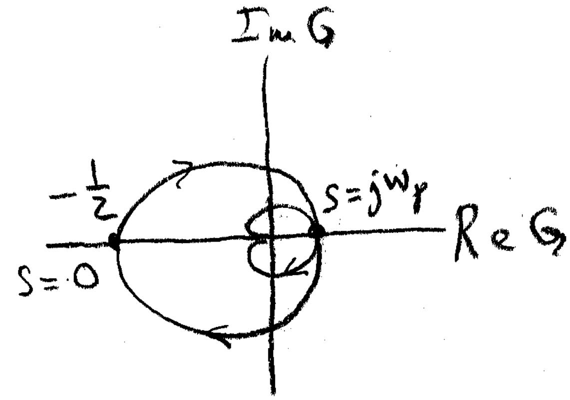
1) Consider the plant

$$G(s) = \frac{s-1}{(s+1)(s^2+2s+2)}$$

under gain compensation.

a) Sketch the Nyquist plot of $G(s)$.





b) Find the phase crossover frequency ω_p of $G(s)$, and evaluate $G(j\omega_p)$.

$$\angle G(j\omega) = \angle \frac{j\omega+1}{j\omega-1} - \angle -\omega^2+j2\omega+2$$

$$= \angle (j\omega-1)(-j\omega+1) - \angle j2\omega+2-\omega^2$$

$$= \angle j2\omega+\omega^2-1 - \angle j2\omega+2-\omega^2$$

$$= 0$$

$$\omega^2-1 = 2-\omega^2 \Rightarrow \omega_p = \sqrt{\frac{3}{2}}$$

$$G(j\omega_p) = \frac{1}{(-\omega_p^2+j2\omega_p+2)} = \frac{1}{\sqrt{(2-\frac{3}{2})^2+4(\frac{3}{2})}} = \frac{2}{5}$$

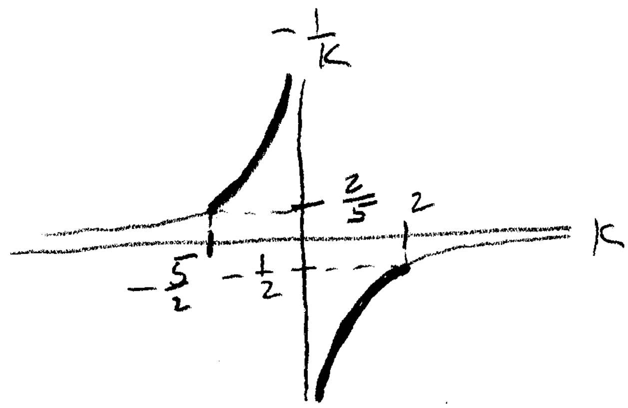
c) Based on the Nyquist plot, find the range of gains K such that the closed-loop system is BIBO stable.

$-\frac{1}{K} < -\frac{1}{2}$ or $-\frac{1}{K} > \frac{2}{5} \Rightarrow p - \gamma = 0 - 0 = 0$ RHP poles

$-\frac{1}{2} < -\frac{1}{K} < 0 \Rightarrow p - \gamma = 1$ RHP pole

$0 < -\frac{1}{K} < \frac{2}{5} \Rightarrow p - \gamma = 2$ RHP poles

H BIBO stable $\Leftrightarrow$ $-\frac{1}{K} < -\frac{1}{2}$ or $-\frac{1}{K} > \frac{2}{5}$

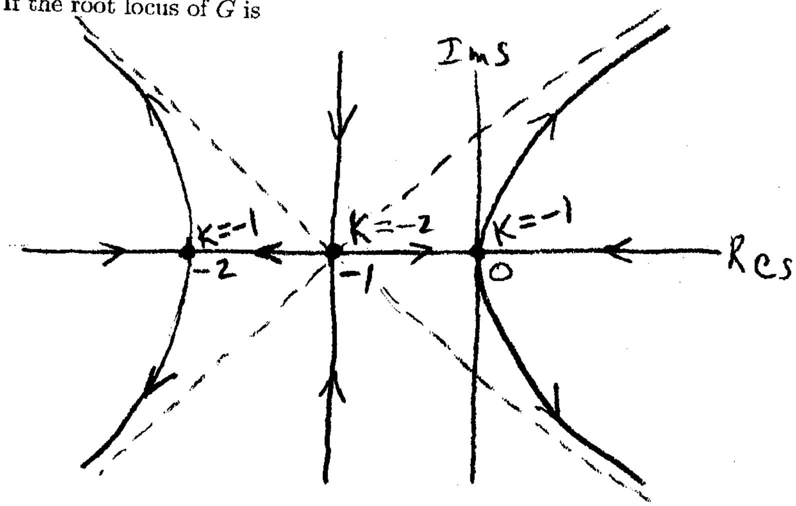


$-\frac{5}{2} < K < 2$

2) Suppose

$$G(s) = \frac{b_m s^m + \ldots + b_0}{s^n + a_{n-1}s^{n-1} + \ldots + a_0}$$

with $m < n$. If the root locus of G is



find

a) n

4 branches $\Rightarrow$ $n = 4$

b) m (Hint: Examine asymptotes.)

8 asymptotes $\Rightarrow 2(n-m) = 8$

$m = 0$

c) $a_1, \ldots, a_{n-1}$ (Hint: Examine saddle points.)

$$G(s) = \frac{b_0}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

$$b_0(4s^3 + 3a_3 s^2 + 2a_2 s + a_1) = 0$$

saddle points at $s = 0, -1, -2 \Rightarrow$

$$s^3 + \frac{3}{4} a_3 s^2 + \frac{1}{2} a_2 s + \frac{1}{4} a_1 = s(s+1)(s+2)$$
$$= s^3 + 3s^2 + 2s$$

$$a_1 = 0, \quad a_2 = 4, \quad a_3 = 4$$

d) a_0 and $b_0, \ldots, b_m$ (Hint: Examine K values.)

$$G(s) = \frac{b_0}{s^4 + 4s^3 + 4s^2 + a_0}$$

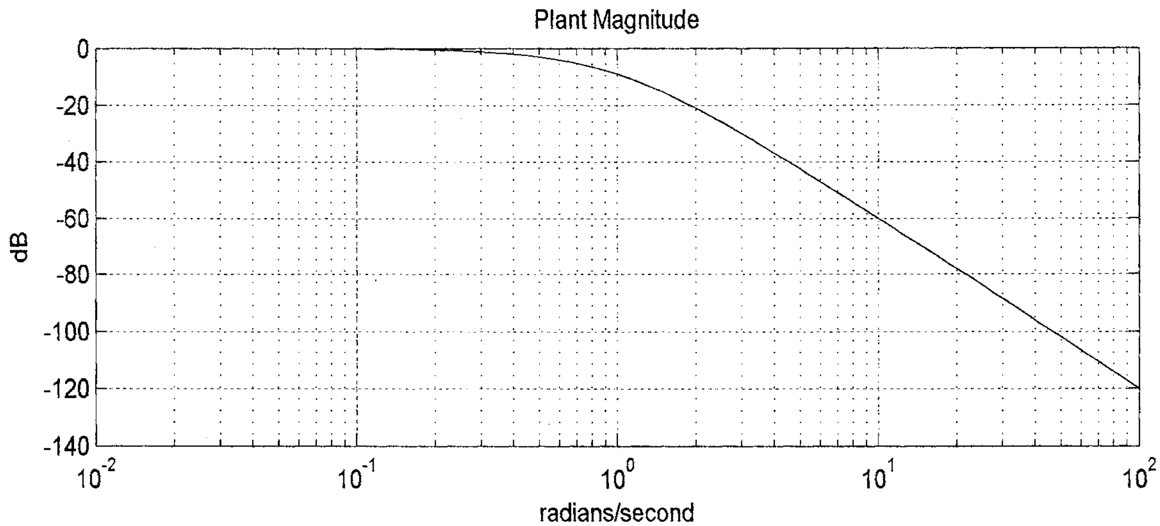
$$D + KN = s^4 + 4s^3 + 4s^2 + a_0 + b_0 K$$

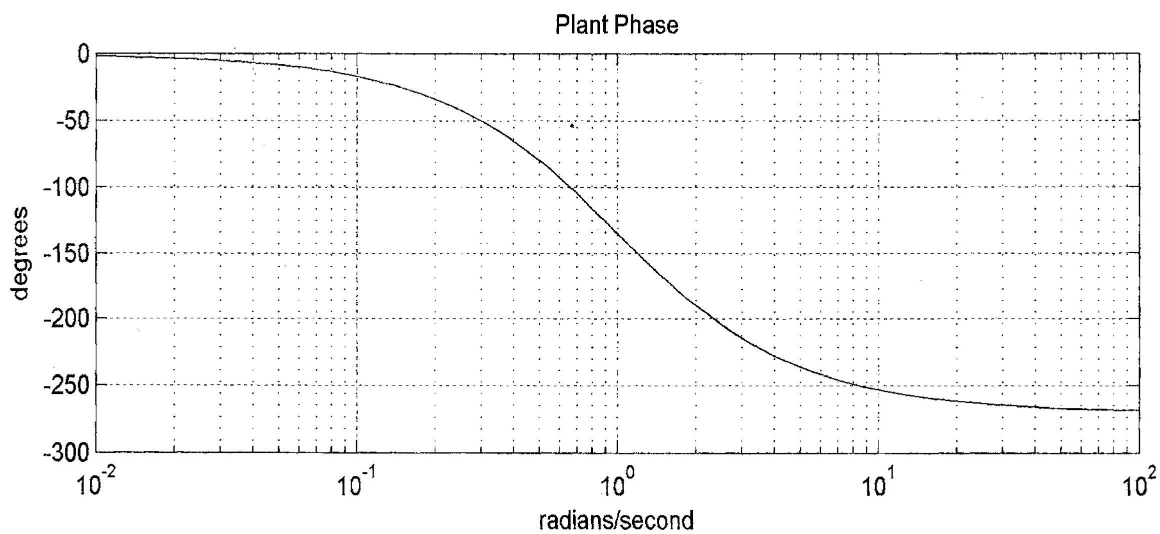
$$D(0) - N(0) = 0 \Rightarrow \quad a_0 - b_0 = 0$$
$$D(-1) - 2N(-1) = 0 \Rightarrow 1 + a_0 - 2b_0 = 0$$

$$a_0 = b_0 = 1$$

$$G(s) = \frac{1}{s^4 + 4s^3 + 4s^2 + 1}$$

3) Consider the plant with Bode plots shown.The plant has no open RHP pole.





Design a lead-lag compensator to achieve the closed-loop specifications

$$essq \leq 0.05$$

$$\phi_p' \geq 60^o$$

$$\omega_b \geq 10 \text{ rad/sec}$$

From the Bode plots, G does not have a pole at $s=0$, so $q=0$. We need

$$K \geq \frac{1-e_{sso}}{e_{sso}|G(0)|} = \frac{.95}{.05(1)} = 19$$

For single-stage lead-lag, we need

$$\phi_\ell = \phi_p' - \angle G(j\omega_g') - 174$$

Setting $\omega_g' = 10$ guarantees the ω_b specification.

$$\phi_\ell = 60 - \angle G(j10) - 174 = 60 + 253 - 174 = 139 > 90$$

At least 2 stages are required. For 2 stages,

$$\phi_\ell = \frac{60 - \angle G(j10) - 180}{2} + 6 = 72.5$$

$$d = \omega_g' \tan\left(\frac{\phi_\ell + 90}{2}\right) = 10 \tan 81.25 = 65.0$$

$$c = \frac{(\omega_g')^2}{d} = \frac{10}{\tan 81.25} = 1.54$$

$$b = \frac{\omega_g'}{10} = 1$$

$$a = \frac{b}{(K|G(j\omega_g')|)^{\frac{1}{2}}\sqrt{\frac{d}{c}}} = \frac{1}{\sqrt{.019}\tan 81.25} = 1.12$$

$$G_c(s) = K\left(\frac{\frac{s}{b}+1}{\frac{s}{a}+1}\right)^2\left(\frac{\frac{s}{c}+1}{\frac{s}{T}+1}\right)^2$$

4) Consider the plant with transfer function

$$G(s) = \frac{s}{s^2 - 1}.$$

Assuming the plant has no hidden modes, design a compensator so that

a) the closed-loop system is asymptotically stable,
b) all loop transfer functions are stable,
c) the closed-loop transfer function has all its poles at $s = -1$.

G is unstable, non-minimum phase (case IV).

$N^+ = s, \; N^- = 1, \; D^+ = s-1, \; D^- = s+1$

$\deg D_{CL} \geq \deg N^+ + \deg D^+ + \mathrm{rel}\, G - 1 = 2$

$D_{CL} = (s+1)^2 = s^2 + 2s + 1$

Bezout identity: $sX + (s-1)Y = 1$

N^+ into D^+: $Q_1 = 1, \; R_1 = -1 \Rightarrow k = 1$

$X_1 = -Q_1 = -1, \; Y_1 = 1$

$X = \frac{X_1}{R_1} = 1, \; Y = \frac{Y_1}{R_1} = -1$

$-D^+$ into XD_{CL}:

$$\begin{array}{c|ccc} 1 & 1 & 2 & 1 \\ & & 1 & 3 \\ \hline & 1 & 3 & \boxed{4} \end{array}$$

$M = -(s+3), \; \tilde{N} = 4$

$YD_{CL} - N^+M = -(s^2+2s+1) + s(s+3) = s-1$

$$G_C = \frac{D^-\tilde{N}}{N^-(YD_{CL} - N^+M)} = \frac{4(s+1)}{s-1}$$

$$H = \frac{N^+\tilde{N}}{D_{CL}} = \frac{4s}{(s+1)^2}$$