ECE 332
Spring 2008
Midterm Exam

Name: Solutions

Important Note: To receive full credit, you must show all your work and fully justify your answers.

1) Determine whether the system described by each of the following differential equations is aymptotically and/or BIBO stable.

a) $\ddot{y} + 2\dot{y} + y = \dddot{u}$

$\Delta(s) = s^2 + 2s + 1 = (s+1)^2 \Rightarrow$ asymptotically stable

$$H(s) = \frac{s^3}{s^2 + 2s + 1}$$

H improper $\Rightarrow$ not BIBO stable

b) $y^{(4)} + 5\dddot{y} + 9\ddot{y} + 7\dot{y} + 2y = u$

$\Delta(s) = s^4 + 5s^3 + 9s^2 + 7s + 2$

$$\mathcal{H} = \begin{bmatrix} 5 & 7 & 0 & 0 \\ 1 & 9 & 2 & 0 \\ 0 & 5 & 7 & 0 \\ 0 & 1 & 9 & 2 \end{bmatrix} \qquad \begin{vmatrix} 5 & 7 & 0 \\ 1 & 9 & 2 \\ 0 & 5 & 7 \end{vmatrix} = 315 - 50 - 49 = 216 > 0$$

asymptotically stable

H proper $\Rightarrow$ BIBO stable

c) $y^{(4)} + 4\dddot{y} + 6\ddot{y} + 4\dot{y} + 65y = \ddot{u} - 2\dot{u} + 5u$

$$\Delta(s) = s^4 + 4s^3 + 6s^2 + 4s + 65$$

$$\mathcal{H} = \begin{bmatrix} 4 & 4 & 0 & 0 \\ 1 & 6 & 65 & 0 \\ 0 & 4 & 4 & 0 \\ 0 & 1 & 6 & 65 \end{bmatrix} \qquad \begin{vmatrix} 4 & 4 & 0 \\ 1 & 6 & 65 \\ 0 & 4 & 4 \end{vmatrix} = 96 - 16 - 1040 < 0$$

not asymptotically stable

$$H(s) = \frac{s^2 - 2s + 5}{s^4 + 4s^3 + 6s^2 + 4s + 65}$$

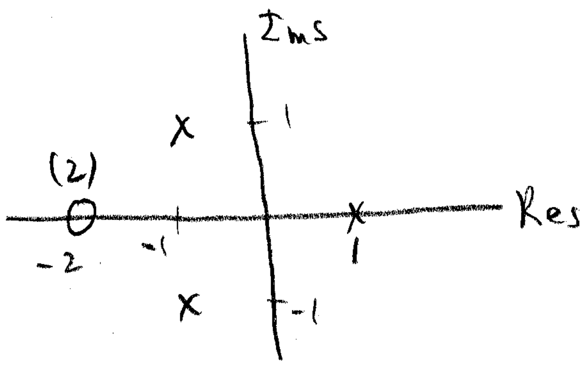
The numerator has RHP roots. If cancellation occurs, we may still have BIBO stability.

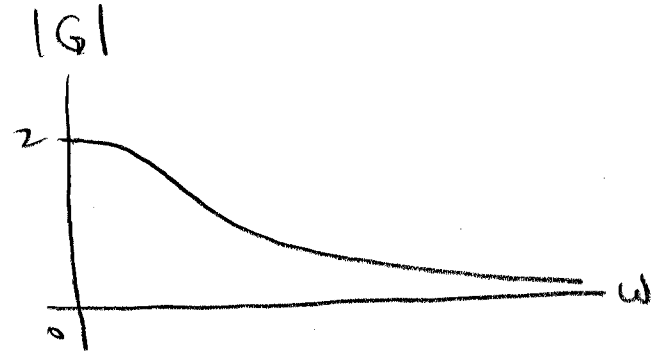
$$\begin{array}{cc|ccccc} 2 & -5 & 1 & 4 & 6 & 4 & 65 \\ & & & & -5 & -30 & -65 \\ & & & 2 & 12 & 26 & \\ \hline & & 1 & 6 & 13 & 0 & 0 \end{array}$$

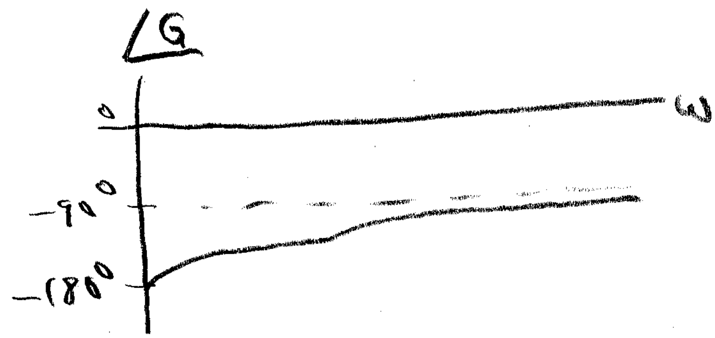
$$H(s) = \frac{1}{s^2 + 6s + 13} \Rightarrow \text{BIBO stable}$$

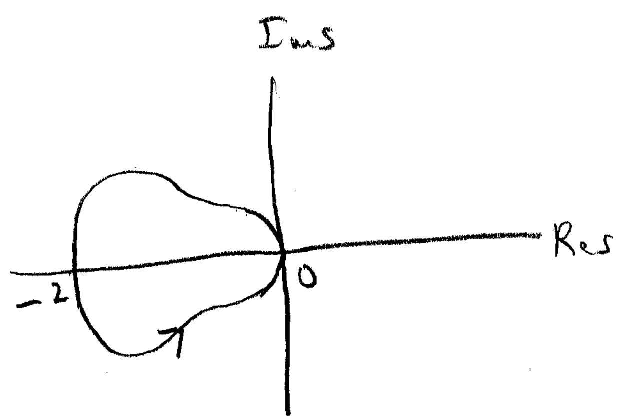
2) For each of the following transfer functions, sketch the magnitude, phase, and Nyquist plots. Be sure to label all important values. Use a linear scale on every axis.

a) $G(s) = \frac{(s+2)^2}{(s-1)(s^2+2s+2)}$

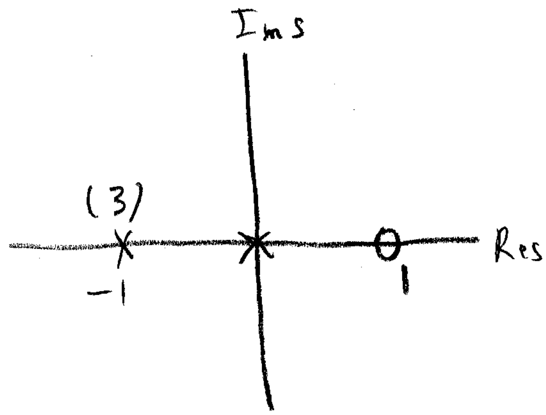


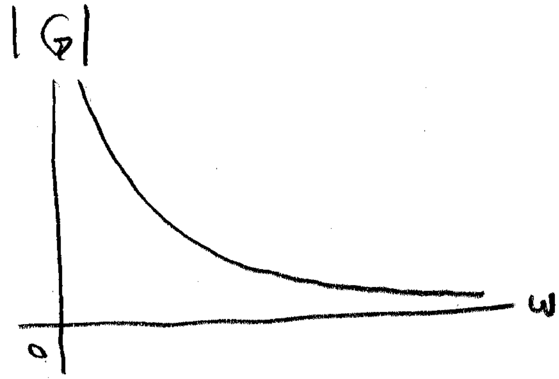


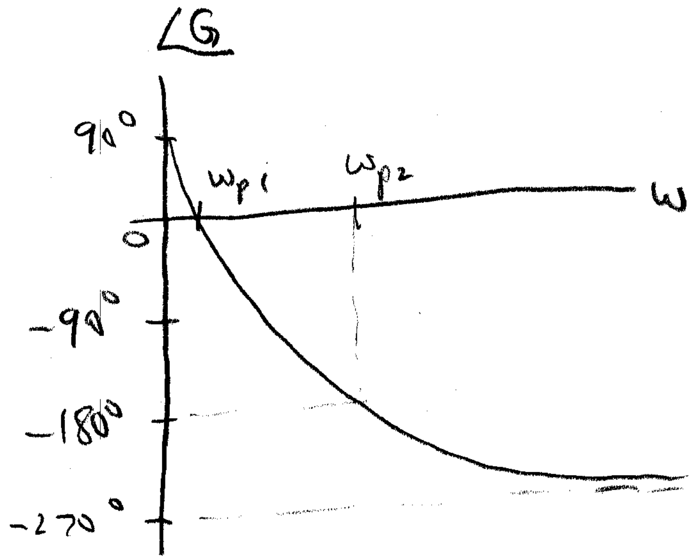




b) $G(s) = \frac{s-1}{s(s+1)^3}$







$$\angle j\omega_p - 1 - 90 - 3\angle j\omega + 1 = 0, -180$$

For any ω,

$$\angle j\omega - 1 = 180 - \angle j\omega + 1 .$$

$$4\angle j\omega_p + 1 = 90, 270$$

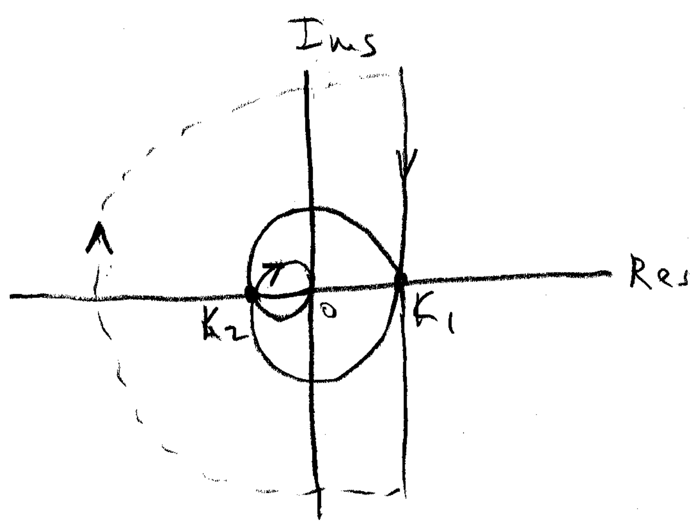
$$\omega_p = \tan 22.5, \tan 67.5$$

$$= \sqrt{2} \mp 1$$

$$K = G(j\omega_p) = \frac{1}{\omega_p(\omega_p^2+1)}$$

$$= \frac{1}{(\sqrt{2}\mp 1)(4 \mp 2\sqrt{2})}$$

$$= \frac{1}{6\sqrt{2} \mp 8}$$



3) For the second-order system

$$H(s) = \frac{3}{s^2 + s + 3},$$

find the

a) natural frequency ω_n

$A = 3, \quad 2\xi\omega_n = 1, \quad \omega_n^2 = 3$

$\omega_n = \sqrt{3}$

b) damping factor ξ

$\xi = \frac{2\xi\omega_n}{2\omega_n} = \frac{1}{2\sqrt{3}}$

c) type number q

$D - N = s^2 + s + 3 - 3 = s(s+1)$

$q = 1$

d) steady-state error e_{ssq}

$P = s + 1$

$e_{ss1} = \frac{P(0)}{D(0)} = \frac{1}{3}$

e) bandwidth ω_b

$$\omega_b = \omega_n\sqrt{1-2\xi^2+2\sqrt{1-\xi^2+\xi^4}} = \sqrt{3}\sqrt{\frac{5}{6}+2\sqrt{\frac{133}{144}}}$$

$$= \sqrt{\frac{5+\sqrt{133}}{2}}$$

f) resonant frequency ω_r

$$\omega_r = \omega_n\sqrt{1-2\xi^2} = \sqrt{3}\sqrt{\frac{5}{6}} = \sqrt{\frac{5}{2}}$$

g) peak resonance M_r

$$M_r = \frac{A}{2\omega_n^2\xi\sqrt{1-\xi^2}} = \frac{3}{\sqrt{3}\sqrt{\frac{11}{12}}} = \frac{6}{\sqrt{11}}$$

h) rise time T_r

$$T_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}} = 2\,\frac{\pi - \tan^{-1}\sqrt{11}}{\sqrt{11}}$$

i) peak time T_p

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{\sqrt{3}\sqrt{\frac{11}{12}}} = \frac{2\pi}{\sqrt{11}}$$