ECE416 HOMEWORK 3

0 Read the textbook sections 3.1 (you can omit example 3.6 if you want, but anyone interested in robotics, navigation, graphics, games, aerospace, or vision applications should read it !), 3.3 up to page 119, and 4.1,4.2,4.3.

1 [evalues and evectors] (a) Write down the definition of an eigenvalue and eigenvector.
(b) Prove that a real symmetric matrix A has all eigenvalues real. (Hint: consider wAv^* where right and left eigenvectors v and w correspond to the eigenvalue λ.)
(c) Give an example of a 2×2 real matrix with complex eigenvalues.

2 [Using evalues and evectors to figure out linear systems] Compute the eigenvalues and eigenvectors of the following matrices. Find the matrix of the corresponding linear map relative to a basis of eigenvectors. Sketch the solution trajectories in $\mathbb{R}^2$ of $\dot{x} = Ax$ for each case indicating the invariant subspaces associated with the eigenvectors. Sketch each case in the original coordinates and (except for case (b)) relative to the basis of eigenvectors.
(a) [sink] $\begin{pmatrix} -3 & 4 \\ -0.75 & -7 \end{pmatrix}$
(b) [spiral sink] $\begin{pmatrix} -2 & -1 \\ 1 & -2 \end{pmatrix}$
(c) [saddle] $\begin{pmatrix} -6 & -4 \\ 4 & 11 \end{pmatrix}$
(d) [non-trivial Jordan block] Do as much of this question as you can in the case $\begin{pmatrix} -2 & 1 \\ 0 & -2 \end{pmatrix}$.
What goes wrong here?
You can check your answers with the Matlab software if you want to.

3 Find the characteristic polynomial of the matrix $A = \begin{pmatrix} 0 & 0 & 0 & -24 \\ 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & -35 \\ 0 & 0 & 1 & 10 \end{pmatrix}$ and the matrix of the corresponding linear map relative to a basis of eigenvectors. Write down a matrix whose characteristic polynomial is $\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$, where a_3, a_2, a_1, a_0 are given real numbers.

4 [what determinants really are geometrically] Let α be a linear map $\mathbb{R}^2 \to \mathbb{R}^2$ with matrix A. Let S be a unit square. Show that $\det(A)$=area($\alpha(S)$). What is the corresponding claim in 3 dimensions and is it true?