# ECE334 HOMEWORK 4

0 [reading the text] sections 4.4.1, 6.1-6.4, 4.4.2, 4.4.4

1 [nilpotent matrices] A matrix N is *nilpotent* if there is a positive integer $k$ such that $N^k = 0$. Let

$$N = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} \qquad \text{and} \qquad S = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Show that $N$ is nilpotent and compute $e^N$ and $e^{St}$. Show that $S$ and $N$ commute and compute $e^{At}$ where $A = S + N$.

2 [theory of linear matrix differential equations] (a) Write down the general solution $x(t)$ to the differential equations $\dot{x} = Ax + Bu$; $x(0) = x_0$. Check that it is indeed the solution.
(b) Solve $\dot{x} = Ax + Bu$, that is find $x(t)$ for $t \geq 0$ where $A$ is given in question 1, $B = (0, 0, 1)^T$, $x_0 = (0, 1, 0)^T$ and $u(t) = 1$ using the matrix methods of the course.

3 Prove that $\frac{d}{dt} e^{At} = Ae^{At}$.
Suppose that $B = P^{-1}AP$ where $A$ is diagonal. What is a simple formula from which $e^B$ can be computed if $P$ were known? What are the columns of $P^{-1}$? (You do not have to compute the numerical values of the columns of $P^{-1}$.)

4 Suppose that the matrix $A$ leaves the subspace $W$ of $\mathbb{R}^n$ invariant (the definition of invariant is that $Aw$ is in $W$ for all $w$ in $W$). Show that $e^A$ also leaves $W$ invariant.

5 [decomposition into modes] Let $A$ be diagonalizable $3 \times 3$ matrix with eigenvalues $\lambda_1$, $\lambda_2$, $\lambda_3$ with corresponding right and left eigenvectors $v_1$, $v_2$, $v_3$, and $w_1$, $w_2$, $w_3$. If $\dot{x} = Ax$ then prove that

$$x(t) = \sum_{j=1}^{3} (w_j x(0)) v_j e^{\lambda_j t}$$

(Remember that we are writing left eigenvectors as row vectors and right eigenvectors as column vectors.)

6 [modes of a circuit] Find the differential equations of the following circuit in terms of the currents $x_1$, $x_2$ shown and find the modes (eigenvectors) and their time constants (related to the eigenvalues).