ECE334 HOMEWORK 6

0 [reading] examples 3.10 and 3.11 [some interestingly analogous discrete time theory] 8.1, 8.2.1, 8.2.4, 8.2.5, 8.3.1

1 [concepts of stability] Let $\dot{x} = Ax$, x in $\mathbb{R}^2$ for this question.
(a) Write down the definition of zero being asymptotically stable. If $A = \begin{pmatrix} -4 & -4 \\ \frac{3}{2} & 1 \end{pmatrix}$, prove that zero is asymptotically stable.
(b) Write down the definition of zero being stable. Give an example of a system that is stable but not asymptotically stable.
(c) Suppose that $||e^{At}|| < 3$ for $t \geq 0$. Prove that the system is stable.
(d) Suppose that $||e^{At}||$ tends to zero as t tends to ∞. Prove that the system is asymptotically stable.

2 [tests for controllability and observability] Check the controllability and observability of

$$\dot{x} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -1 \end{pmatrix} x + \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} u$$

$$y = [1\ 0\ 1]x$$

3 Suppose the system $\dot{x} = Ax + bu$ has state x in $\mathbb{R}^3$ and a single input u and is controllable. A is 3×3 and b is 3×1. Let the characteristic polynomial of A be $\lambda^3 + \alpha_2\lambda^2 + \alpha_1\lambda + \alpha_0$. What are the A' and b' matrices for the system if we change to a new basis $\{b, Ab, A^2b\}$? Why do we need to assume that the system is controllable?

4 Consider the 2 carts connected by a spring and driven by 2 motors in homework 5 problem 5.
(a) Is it controllable using only one motor?
(b) Is it controllable using both motors?
(c) Is it observable if only the position z_1 of the first car is measurable?
(d) Is it observable if only the velocity v_1 of the first car is measurable?
(e) Is it observable if the velocities v_1, v_2 of both cars are measurable?

5 Consider $\dot{x} = Ax + Bu$; $y = Cx$.
(a) If the basis is changed according to $x = Tx'$, what are the new matrices A', B', C'?
(b) Show that the characteristic polynomial does not depend on which basis is used to compute it.
(c) Show that the controllable subspace does not depend on the basis used to compute it.

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$$\dot{x} = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -2 & 2 \\ 0 & 0 & -3 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} u$$

$$y = [0\ 1\ 0]x$$

(a) Determine whether the system is controllable and observable using the Q and Q' matrix tests.
(b) Find the controllable and unobservable subspaces.
(c) Diagonalize the system (change coordinates to diagonalize the A matrix and compute the new b and c vectors). Now determine which individual modes are controllable and/or observable.
(d) Find the transfer function of the system and observe which system modes appear as system poles. (Be sure to make any possible cancellations in the transfer function.)

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8 [computing controls]
(a) Write down the formula for the controllability Gramian M in terms of the (A, B, C) matrices. The input is $u(t)$. State the formula for the input $u(t)$ that drives the system state $x(0) = 0$ to a given state $x(T)$ at time T. Show that your formula works. You should assume the system is controllable and you should point out where you use this assumption.
(b) Check the controllability of the following system:

$$\dot{x} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u$$

(c) Compute a control $u(t)$ which takes the system state from the origin at time 0 to $(2, 1)^T$ at time 2.
(d) Compute a control $u(t)$ which takes the system state from $(2, 1)^T$ at time 2 to $(3, 3)^T$ at time 4.
(d) Use Matlab commands ss and lsim to check your answers to (c) and (d).

9 problem 8.9 in the Bay text