ECE334 HOMEWORK 7

0 [reading] Sections 10.1 and 10.3 through Example 10.4.

1 [controllability and pole placement]

$$\dot{x} = \begin{pmatrix} -1 & 4 \\ -1 & 3 \end{pmatrix} x + \begin{pmatrix} 3 \\ 2 \end{pmatrix} u$$

$$y = [-1 \ \ 3]x$$

(a) Show that the system is controllable.[Note that it is possible for a *nondiagonalizable* matrix to have repeated eigenvalues and still be the A matrix of a controllable system; what we showed in class was that a *diagonalizable* A matrix with repeated eigenvalues gave a noncontrollable system]
(b) Put the system into controllable canonical form and find the transformation T to controllable canonical form coordinates x' so that $x = Tx'$.
(c) What is the input output transfer function of the system?
(d) Find state feedback gains $k = (k_1, k_2)$ which places both poles of the closed loop system at -1.
(e) Compute the closed loop eigenvalues to check the answer to (d).

2 [stabilizability and pole placement]

$$\dot{x} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ -1 \end{pmatrix} u$$

(a) Find a basis of the controllable subspace and show that the controllable subspace is invariant under the A matrix.
(b) Show that the system is stabilizable.
(c) Find state feedback gains $k = (k_1, k_2)$ which place both poles of the closed loop system at -1.
(d) Compute the closed loop eigenvalues to check the answer to (c).

3 Consider the 2 carts connected by a spring and driven by 2 motors in homework 5 problem 5.
We want to bring the two carts to rest at the origin using only the motor on cart 1. Find the gain matrix k in the control law $e_1 = kx$ which places the poles at $-1 \pm j, -100 \pm 100j$.

4 [reachability and controllability]
(a) Show that the reachable subspace is invariant under the A matrix.
(b) Argue using (a) that if x_1 is a reachable point then x_1 is a controllable point.