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ANALYSIS OF BURSTING IN TELEPHONY LOOPS WITH ADAPTIVE HYBRIDS

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ABSTRACT

Adaptive filtering techniques have been successfully applied to attenuate echoes in long distance telephone connections. Recently, with the increased ability to process signals cheaply, adaptive echo cancellation is being employed on shorter telephone circuits. To avoid the possibility of entrapment in an echo canceller initialization that causes singing, doubletalk detectors cannot be used aggressively on such highly variable short connections. While echo cancelers do tend to be effective on the shorter circuits, in the absence of doubletalk detectors a new (and undesirable) phenomenon called bursting has been observed in real-time laboratory tests of such applications. Bursting is characterized by long periods of successful echo attenuation alternating with short periods of wildly oscillating signals. By analyzing a simplified abstraction of this problem, this paper presents a theoretical explanation of the kernel of the bursting problem: an imbalanced excitation combination. The structural source of the problem is the use of an adaptive filter in a feedback loop.

1. INTRODUCTION

"Echo loves Narcissus." - Grecian graffiti

The fundamental problem addressed by echo control in telephone systems is illustrated in Figure 1. Inevitably, there is a mismatch in the impedance characteristics of the two wire loop and the balancing network of the hybrid circuit, and some energy from the received signal is returned in the transmitted signal. The application of adaptive filtering technology to remove a significant proportion of this echo has been so successful that it appears in recent introductory texts on adaptive filtering, e.g. [1] and [2], as prime evidence of the utility of adaptive filtering. An adaptive hybrid incorporating echo cancellation is illustrated in Figure 2.

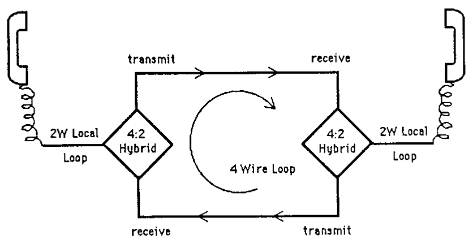

Figure 1: The Echo Problem in Telephone Systems

The generation of the echo can be represented by passing the received signal x_k through a linear time invariant filter with a finite impulse response (FIR) h as in Figure 3. Two signals are added to the echo before the sum y_k is returned to the adaptive echo canceler. A "noise" signal n_k encompasses the part of the echo which cannot be captured by the FIR filter representation, of the echo path. The signal v_k represents the speech signal of the near end speaker which should be allowed to pass through the hybrid and canceler undisturbed. The adaptive hybrid attempts to remove the part of y_k which is significantly correlated with x_k within the time windows set by the length of the adaptive FIR filter.

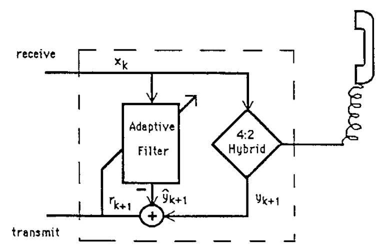

Figure 2: Basic Adaptive Hybrid

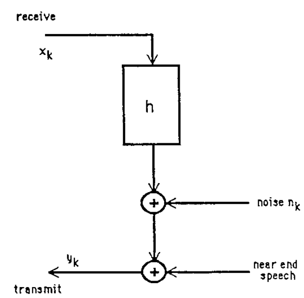

Figure 3: Model of the 4:2 Hybrid (Echo Path)

Existing theory suggests certain operating conditions under which such an adaptive echo canceler should perform well. Theoretical investigations of adaptive filters demonstrate desirable behavior (without their encasement in a feedback loop such as in Figure 4) when driven by "persistently exciting" inputs [3] and when disturbances are small. Thus the adaptation process benefits when the far-end input contains a frequency range of components well-distributed enough to persistently excite the adaptive system at the near end and the interfering signals v_k and n_k are small. These conditions are fairly closely met in echo canceler implementations on long (if not short) distance connections. For example, sufficiently rich excitation is usually provided by speech signals. In common practice, doubletalk detectors turn off adaptation when v_k is too large relative to x_k. And adaptive filter lengths are chosen (in part) to keep n_k small.

Another approach to the analysis of adaptive systems relies on decorrelation of the "inputs" from the "disturbances" [4]. In the echo

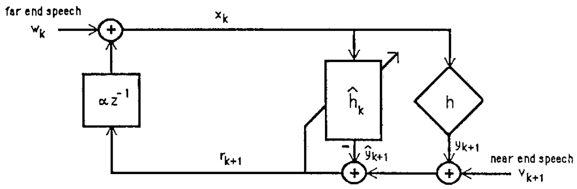

Figure 4: Single Adaptive Hybrid

cancellation context, this would require that x be generated independently from v. Yet (due to the feedback nature of the communication loop) the received signal (x) at the right hand canceler of Figure 1 invariably contains a component which represents the residual echo of its own transmission (y) reflected through the hybrid on the left hand side. Since the transmission (y) includes the speech (v) at the right hand canceler, there is a source of correlation between x and a sequentially correlated v, which is essentially due to a feedback of the transmitted signal. When the length of the delay introduced in the four wire path is long enough and the signals themselves are broad band enough, this source of correlation is benign, since it is outside the time window of the adaptive filter. The effects of the correlation, however, become more pronounced as the length of the connection (and hence the length of the delay) is decreased.

In an effort to further automate echo control and to comply with prescribed echo loss requirements in connections to common carriers, adaptive filters such as the adaptive hybrid of Figure 2 are being used in many short connections. Such short connections do not provide the decorrelation delay of long distance lines, so x and v can be significantly correlated, especially when v is present and w is not. Given the possibility of such situations, predictions of robust behavior cannot be based on a persistent excitation and small disturbance combination [5] or on a near-decorrelation of the excitation and disturbance [6]. However, the persistent excitation and small disturbance scenario can be enforced using doubletalk detectors, which can switch adaptation on and off. Assume that adaptation has progressed to the point that effective, but not necessarily perfect, echo cancellation is being achieved. If w is also small, then x is small. By our adequate excitation and small disturbance scenario, if v exceeds a modest threshold, adaptation should be halted. In other words, in this absence of doubletalk (i.e. v present but w effectively absent), adaptation should not take place for fear of nonrobust, potentially catastrophic behavior. As noted earlier, simply testing the ratio of the energy in v to the energy in x against a threshold is a common method of implementing what is called a doubletalk detector.

The special feature of the adaptive hybrid is that it does not operate in open-loop, as suggested by Figure 2. This point was recognized in our preceding discussion of the potential correlation of v and x. The adaptive hybrid is more appropriately analyzed as an element in a feedback loop, as illustrated for a simplistic case in Figure 4. In Figure 4, the 4W loop signal path through the near end hybrid is modeled simply as a gain h. The adaptive echo canceler is similarly modeled as a simple gain $\hat{h}$, albeit adjustable. The echo return path is modeled as αz^{-1}, which appropriately includes a gain and a short delay. Treating this loop as a feedback system with two inputs, v and w, and two outputs, r and x, yields a characteristic equation of $1 - \alpha(h-\hat{h})z^{-1}$ for a fixed $\hat{h}$. Assume that $\hat{h}$ is fixed by the cessation of adaptation by the doubletalk detector at the near end, because v is too big. If $|\alpha(h-\hat{h})| > 1$, the resulting time-invariant feedback loop is unstable. Imagine switching into a circuit with a nominal $\hat{h}$ but a worst case (α,h) pair. Conceivably, if $|\alpha(h-\hat{h})| > 1$ and the doubletalk detector is immediately engaged by the message traffic, this initial instability will not be corrected before the circuit experiences what is referred to as "singing". To avoid entrapment in a singing mode, doubletalk detectors should be used conservatively. Conservative use means that v must be quite large, or the SNR of the "signal" x to noise v must be quite small, before adaptation is halted.

What happens if the doubletalk detector is removed and x is substantially correlated with v within the filter length window of the adaptive FIR echo canceler? A new phenomenon, called bursting, has been observed in experimental tests at Tellabs Research Laboratory specifically designed to examine such cases [7]. Long periods of close match between the output of the adaptive filter and the echo path during which the echo canceler appears to be functioning well, suddenly (and with no apparent warning) degenerate into wild oscillation, which then restabilizes just as suddenly. Real time laboratory tests at Tellabs utilized a 20 tap adaptive hybrid at the near end of a line and a simple (nonadaptive) hybrid at the far end. With independent narrowband modem signals of approximately equal amplitude at each end, and with a nonadaptive echo attenuation of about -6 db, such bursts appeared intermittently. When the transmission at the far end was quiescent, the bursts appeared more frequently. In this latter case, we clearly fail our desire for the adequate excitation and small disturbance combination and its accompanying robustness that excludes bursting behavior. It is this latter case of extremely imbalanced excitation with w zero and v nonzero in conjunction with the absence of doubletalk detectors that we will consider as a prototype for uncovering the source of this experimentally observed bursting.

This paper characterizes this experimentally observed bursting (mis)behavior as long periods of good echo attenuation (during which the parameters of the adaptive mechanism slowly drift towards a setting destabilizing the closed-loop system) alternating with brief periods of rapid oscillation of signals throughout the system (during which the adaptive mechanism quickly restabilizes). Figures 5 and 6 show such bursting in simple simulations of adaptive hybrids. This bursting of the adaptive hybrid is closely related to "bursting" in adaptive control [8] in that it is the result of underexcitation and disturbances combined with the existence of a feedback path around the adaptive mechanism. The parameter drift of the quiescent phase is similar to the parameter drift observed in the underexcited LMS adaptive filter in [9]. The recovery from the burst is closely related to the "self-stabilization" property of [10] in which the adaptive algorithm generates an exponentially growing output error (the burst) which it then uses as a form of spectrally rich, self-generated excitation in order to regain stability.

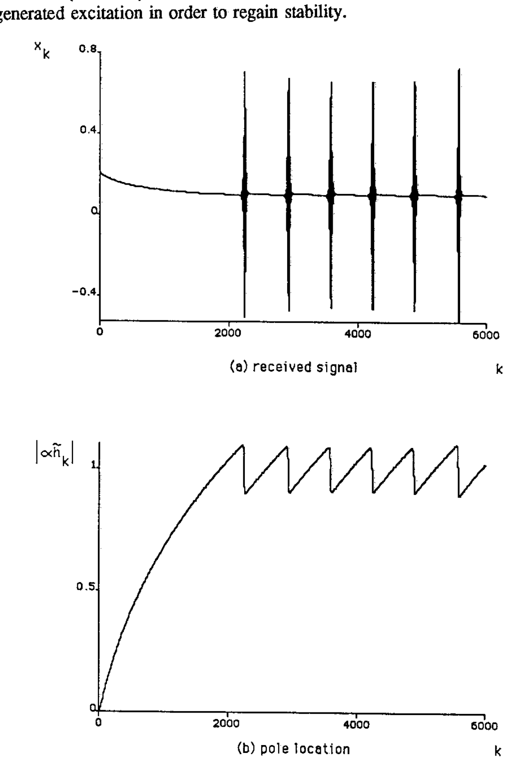

Figure 5: Bursting with Constant Near End Transmission

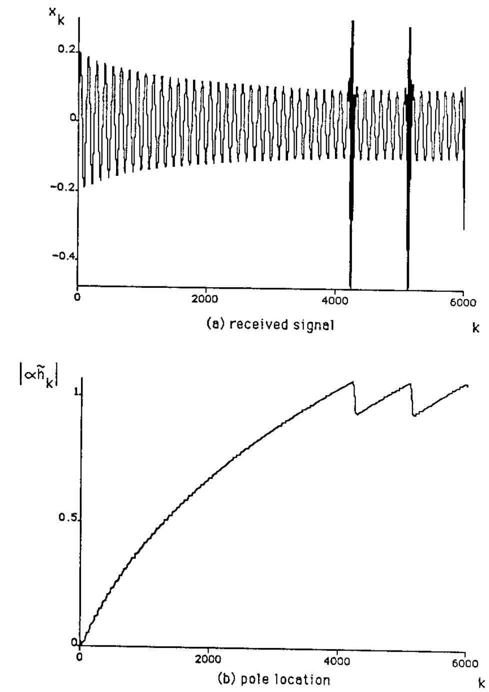

Figure 6: Bursting with Sinusoidal Near End Transmission

As observed, bursting is especially frequent when there is a narrowband signal transmitted from the adaptive end but the nonadaptive end is quiescent. This paper analyses this situation in a suitable abstract form (à la Figure 4) in section 3, and reveals that during the quiescent period the parameters of the adaptive mechanism are "winding up" or "drifting," and are eventually pushed across the stability boundary of the closed loop feedback system. Signal growth follows, and then the system restabilizes. Section 2 introduces the mathematical setup of the echo canceler and the models which will be used to demonstrate the bursting mechanism in section 3.

2. PROBLEM STATEMENT AND OVERVIEW

Our model for studying the salient features of adaptive echo cancelers on phone lines is diagrammed in Figure 4. The adjustment of the adaptive parameters, represented by the arrow through the $\hat{h}$ box in Figure 4, uses the error $y_k - \hat{y}_k$ ("corrupted" by v_k) to adjust the $\hat{h}$ parameter with the LMS parameter update law

$$\hat{h}_{k+1} = \hat{h}_k + \mu\, x_k\, r_{k+1} \tag{2.1}$$

where μ is the stepsize,

$$r_{k+1} = y_{k+1} - \hat{y}_{k+1} + v_{k+1} \tag{2.2}$$

is the error sequence,

$$y_{k+1} = h\, x_k \tag{2.3}$$

is the near end echo source,

$$\hat{y}_{k+1} = \hat{h}_k\, x_k \tag{2.4}$$

is the output of the adaptive filter, and

$$x_k = \alpha\, r_k + w_k \tag{2.5}$$

is the echo from the far end. In (2.5), α is a small positive constant representing the attenuation of the echo and w_k is the signal transmitted from the far end. Combining (2.1) to (2.5) and introducing the parameter estimate error variable $\tilde{h}_k = h - \hat{h}_k$, produces

$$\tilde{h}_{k+1} = (1 - \mu\, x_k^2)\, \tilde{h}_k - \mu\, x_k\, v_{k+1} \tag{2.6}$$

$$x_{k+1} = \alpha\, \tilde{h}_k\, x_k + \alpha\, v_{k+1} + w_{k+1}. \tag{2.7}$$

Typical operation of the system supposes that the near and far end transmissions v_k and w_k alternate frequently, which helps to ensure that the algorithm is adequately excited. Indeed, if w_k and v_k are relatively uncorrelated and if w_k is persistent (so x_k is persistent), then (2.6) is exponentially stable and hence is convergent to some ball about $\tilde{h} = 0$ (equivalently, $\hat{h}$ is nearly equal to h, implying that the echo is suppressed). This statement relies on the persistent excitation and small disturbance combination used in [5].

The bursting phenomena we are investigating appears when the transmission at the far end is quiescent while the transmission at the near end is active. This situation will be investigated by making the simplifying assumptions that $w_k = 0$ and $v_k = 1$ for every k. This situation is simulated in Figure 5, which shows large bursts after 2000 iterations of good behavior. The situation when v_k is sinusoidal results in similar behavior as shown in Figure 6. In outline, the bursting phenomena can be explained in the following three steps.

(A) Suppose $|\alpha\tilde{h}| < 1$, and that x_k is small. Then (2.7) is a stable linear system, and (2.6) is a victim of the "competing rate" phenomena as in [9]; the contracting term $(1 - \mu x_k^2)$ tends to reduce the magnitude of $\tilde{h}$ while the driving term $-\mu x_k v_{k+1}$ tends to increase the magnitude. For small x_k, the latter effect dominates, and $\tilde{h}$ experiences a slow (linear) drift. This drift continues until $|\alpha\tilde{h}| > 1$.

(B) When $|\alpha\tilde{h}| > 1$, (2.7) is unstable and x_k begins to grow exponentially. This causes the contraction term in (2.6) to dominate the driving term, which implies that $\tilde{h}$ contracts exponentially. This is essentially the self-stabilization of [10].

(C) Soon $|\alpha\tilde{h}|$ becomes less than 1, and x_k itself begins to contract. Eventually, x_k becomes small, returning to the situation in (A).

Consequently, the sequence of events (A) implies (B) implies (C) is not a one time occurrence, but is recurrent. The repetitive destabilization and restabilization of (2.7) appears to be an intrinsic property of the equation pair (2.6) and (2.7) when they are underexcited (e.g., when $w_k = 0$).

3. ANALYSIS OF A SIMPLE MODEL OF THE SINGLE ADAPTIVE HYBRID SYSTEM

This section examines the signal-imbalance-dependent stability properties of a single parameter adaptive echo canceler implemented at the near end of a phone line, as abstracted from Figure 4 in (2.6)-(2.7). The "normal" situation is when the transmissions from both the near and far ends are intermittently present. Under fairly mild excitation conditions, the adaptive echo canceler is provably stable [11].

To elaborate, consider the case when w_k is adequately excited, i.e. a nonzero value of magnitude greater than some positive constant occurs in every t-length window. Furthermore, assume that v_k is small, that $\hat{h}$ almost matches h (so that n_k is nearly zero), and that the system remains stable. In particular, it is possible to guarantee that $|\alpha\tilde{h}_k| < 1$ for all k given a sufficiently small $\overline{v^*} > v_k$ for all k, which implies that the bursting behavior described below cannot occur.

One weakness of such a statement is that it gives no precise measure of the magnitudes involved. The following example explores this in one simple situation.

EXAMPLE: Suppose $w_k = w > 0$ and $v_k = v > 0$ for all k. If $w > \alpha v$, then (2.6)-(2.7) has a locally stable equilibrium at $\tilde{h}^* = -\dfrac{v}{w}$ and $x^* = w$.

Thinking of w as the degree of excitation, and v as the power of the disturbance, this example suggests that the relation $w > \alpha v$ is somehow fundamental to the proper operation of the adaptive hybrid. Indeed, when the inequality is violated, then the "equilibrium" $\tilde{h}^* = -\dfrac{v}{w}$ causes $|\alpha\,\tilde{h}^*| = |\dfrac{\alpha v}{w}| > 1$, which implies that (2.7) is unstable. Such instability is at the heart of the bursting phenomena.

With the typical conversational situation, periods of large w/v are followed by ones with small w/v. Bursting arises when the far end of the phone line is quiescent and the near end is active, i.e. when w/v is small for too long. The simplest way to model this situation is to suppose that

$w_k = 0$ and $v_k = 1$ for every k. This certainly violates the hypotheses of the example. The system (2.6) - (2.7) then becomes

$$\tilde{h}_{k+1} = (1 - \mu x_k^2)\, \tilde{h}_k - \mu x_k \tag{3.1}$$

$$x_{k+1} = \alpha \tilde{h}_k x_k + \alpha, \tag{3.2}$$

which has no finite stationary points. Solving (3.1) for $\tilde{h}^* = \tilde{h}_{k+1} = \tilde{h}_k$ yields $\tilde{h}^* = -\frac{1}{x^*}$. Plugging this into (3.2) gives $x^* = 0$. Thus, as x converges towards its "equilibrium" at 0, $\tilde{h}$ tries to converge to its "equilibrium" at ∞. Eventually, $\tilde{h}$ grows large enough to destabilize (3.2). This is one interpretation of the origin of the bursting phenomenon.

Equations (3.1) and (3.2) are easily simulated. The values of parameters in the simulation of Figure 5 were chosen conservatively. The echo attenuation factor is $\alpha = 0.2$, the echo path at the near end is $h = 0.1$, and the estimator was initialized at $\hat{h}_0 = 0$ with a stepsize of $\mu = 2^{-5}$. Figure 5(a) shows the received signal x_k versus time, while 5(b) plots $|\alpha\tilde{h}_k|$ versus time. The quantity $\alpha\tilde{h}_k$, for satisfactorily slow $\tilde{h}_k$, can be interpreted as the "instantaneous" pole of (3.2). The first burst occurs shortly after 2000 iterations, and then the bursts recur approximately every 600 iterations.

Analysis in [11] of the bursting cycle distinguishes four phases:

(1) a long linear drift of $\tilde{h}$ coupled with a slow decay of x causing $|\alpha\tilde{h}|$ to grow larger than 1 (at time t_1), which is verified by the by the competing rate lemma [11].

(2) The resulting instability after time t_1 causes x to expand rapidly, until x is larger than α at time t_2. The contraction term of the competing rate lemma [11] dominates, and $\tilde{h}$ begins to shrink.

(3) At time t_3, $|\alpha\tilde{h}|$ becomes less than unity, restabilizing (3.2), and finally,

(4) x decays, until at time t_4, the situation has returned to step (1).

Due to the (relative) simplicity of (3.1)-(3.2), this bursting cycle can actually be proven to exist [11].

Simulations of (2.6)-(2.7) with a sinusoidal v_k and zero w_k also show bursting. With α, h, μ, and $\hat{h}_0$ as in the previous simulation and $\omega = 0.05$, Figure 6(a) shows the received signal x_k while 6(b) shows the pole location. The first burst occurs at about 4000 iterations, and the average time between bursts is longer than in the previous case. Fundamentally, however, the behavior is nearly indistinguishable from the bursting of (3.1)-(3.2). The differences are fairly subtle; the long drift of $\tilde{h}$ scallops (unfortunately, not visibly in Figure 6) instead of increasing linearly, the drift of $\tilde{h}$ is driven by the "balanced rate" lemma (lemma 2 of [11]) instead of by the "competing rate" lemma (lemma 1 of [11]), and x becomes a sine-like waveform instead of a long slow decay. Nevertheless, the same basic sequence of events occurs to cause a very similar bursting phenomenon, as proven in [11].

Although the analysis of bursting in [11] does not extend directly to the situation where v_k is characterized by certain stochastic properties, it does provide a framework from which to draw hypotheses. Suppose, for example, that v_k is an independent process. Then there is no correlation between x_k and v_{k+1}. Thus the driving term is zero mean, and drift is unlikely. On the other hand, if v_k is dependent, then x_k and v_{k+1} are correlated. The competing rate idea suggests that this would drive $\tilde{h}$ towards instability. If this drive is substantial, then bursting will result. Simulations with white gaussian v_k did not exhibit bursting, while simulations with colored gaussian v_k (white noise passed through a single pole filter) did exhibit the bursting effect.

One common modification to adaptive algorithms like LMS is the addition of a leakage factor. In the linear case (with no feedback of the error into the output), leakage provides a safety net that guarantees exponential stability of the error system. In the echo cancellation application, however, leakage cannot always prevent bursting. The parameter update with leakage $\lambda \in [0,1)$ replaces (2.6) with

$$\tilde{h}_{k+1} = (1 - \lambda - \mu x_k^2)\, \tilde{h}_k - \mu x_k v_{k+1} + \lambda h. \tag{3.3}$$

For the special case $v_k = 1$ and $w_k = 0$, the system (2.7) and (3.3) has an equilibrium at

$$\tilde{h}^* = \frac{\lambda h - \mu x^*}{\lambda + \mu x^{*2}} \quad ; \quad x^* = \frac{\alpha}{1 - \alpha \tilde{h}^*}. \tag{3.4}$$

Since $\lambda + \mu x^{*2}$ is positive, there are finite solutions to (3.4) for small α. For properly sized λ (such that $|\alpha\tilde{h}^*| < 1$), bursting is unlikely. For leakage too small, however, the continuity of (3.4) shows that the solution must approach the solution for $\lambda = 0$, which is $x^* = 0$, $\tilde{h}^* = \infty$. For such λ, the equilibrium causes (2.7) to be unstable, and bursting will result. Simulations verify that properly scaled λ inhibits bursting, while λ too small allows bursting to occur.

4. CONCLUSION

Bursting was observed in laboratory tests of adaptive hybrids without doubletalk detectors. This bursting was reproduced in simulations in a simplified setting which was then analyzed. (Refer to [11] for the details of this analysis.) The kernel of the bursting phenomenon in echo cancellation lies in a driving term which causes the parameter estimates of the adaptive hybrid to drift linearly, until they eventually destabilize the closed loop system, causing the burst. The contractive power of the parameter estimator then dominates and the system is restabilized. Following restabilization, drift begins again and the bursting cycle can be repeated. The structural source of the problem lies in the use of an adaptive filter in a feedback setting, such that signal growth (and potential unboundedness) can occur for finite-valued, slowly-varying parameter estimates in certain ranges. The situational source of this problem is an extended phase of improperly imbalanced excitation.

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