Sensor Placement for On-orbit Modal Identification of Large Space Structure via a Genetic Algorithm

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Abstract: A variant of the Genetic Algorithm is used to place sensors optimally on a Large Space Structure for the purpose of modal identification. The selection and reproduction schemes of the Genetic Algorithm are modified and a new operator called forced mutation is introduced. These changes are shown to improve the convergence of the algorithm and to lead to near optimal sensor locations. Simulated results are also compared with previous results obtained by the Effective Independence method. The Genetic Algorithm based sensor configuration estimates the target mode response more accurately.

1. INTRODUCTION

On-orbit system identification is a vital component to the successful operation of Large Space Structures (LSS). The number of sensors used on the LSS for system identification is limited due to weight and cost considerations. The sensors must be placed in an optimal fashion so that the modal characteristics of the LSS can be identified as accurately as possible. To determine the optimal sensor locations, extensive prelaunch analysis must be performed since the sensors cannot be easily moved on orbit.

A vast amount of literature has been produced dealing with sensor placement for system identification and control. Much of the work deals with distributed parameter systems. [1]-[5] A relatively small number of papers [6]-[7] have considered sensor placement for structural parametric identification. In [8], the sensor placement problem is considered from the standpoint of a structural dynamicist who must use the data collected from the sensors to validate an LSS Finite Element Model (FEM) using test-analysis correlation techniques as in [9].

In [8], an efficient method called Effective Independence (EfI) is presented to place a small number of sensors for modal identification of an LSS. Based on the prelaunch FEM, a set of target modes is selected for identification. The target mode partitions must be linearly independent or correlation methods, which compare test and analysis mode shapes, will not be able to spatially differentiate between the modes. The EfI method casts the linear independence problem in the form of a target mode response estimation problem. The sensor locations which produce the best target mode response estimate also produce linearly independent target mode shape partitions. An initial candidate set of sensor locations is selected. The approach then ranks the candidate locations based on their contribution to the linear independence of the corresponding FEM target mode partitions. Locations which do not contribute are removed from the candidate set. In an iterative manner, the initial candidate set of sensor locations is reduced to the number of available sensors. The EfI method is efficient yet sub-optimal. The errors between the real target mode response and the estimated target mode response obtained by sensors placed at the locations determined by the EfI method cannot be guaranteed to be minimum.

In this paper, the GA will be applied to the same sensor placement problem presented in [8]. Since the GA is a globally optimal method, the sensor locations estimated by the GA can find the global minimum of the error between the real target mode response and the estimated response. To apply the GA to the problem of sensor placement for modal identification of an LSS, each of the possible sensor locations is represented as an integer (the "gene") and k such sensor locations are concatenated into an integer string (the "chromosome"). Each chromosome represents a possible set of sensor locations. The fitness of each chromosome is defined as the determinant of the corresponding Fisher Information Matrix (FIM).[12] Thus, the quality of any given set of sensor locations is readily discernable. The goal of the GA is to find the "best" chromosome, that is, the set of sensor locations which maximizes the determinant of the FIM.

It will be shown in this paper that the parent selection and reproduction strategies in [10]-[11] hinder the GA from finding the optimal sensor locations for modal identification of the LSS. Both strategies will be modified so that the GA can lead to an optimal solution. A new operator called *forced mutation*, which is performed on any chromosome with redundant sensor locations, is introduced. Mathematically, it is shown that forced mutation improves the convergence of the algorithm.

2. PROBLEM STATEMENT

Due to weight and cost considerations, only a small number of sensors can be placed on the LSS for modal identification. The set of target modes is assumed to include all modes which are strongly excited by the actuator configuration. The set of candidate locations for sensor placement is chosen large enough such that all of the important dynamics within the target modes are included. Assume that there are m candidate locations on the LSS and n target modes to be identified. Let $y \in R^{m\times 1}$ be the vector containing sensor outputs at all the possible candidate locations. Let $\Phi \in R^{m\times n}$ be the matrix of FEM target modes partitioned to the sensor locations; and let $q \in R^{n\times 1}$ be the vector of target modal coordinates. Then,

$$y = \Phi q + w, \qquad (1)$$

where w denotes the vector of measurement noise which is assumed to be zero-mean Gaussian noise.

The set of candidate locations is chosen so that the column vectors of Φ are linearly independent. This spatial independence must be maintained by the final selected sensor configuration. Linear independence of the target mode

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partitions implies that at any time t, the outputs of the sensors can be sampled and the target mode dynamic response can be estimated. Theoretically, only n sensors are needed to identify n target modes. Practically, the number of sensors placed on the LSS must be more than n to account for engineering uncertainty.

Suppose k sensors are to be placed on the LSS. Then, for the i-th set of the selected k sensor locations, the outputs at these sensors are described by

$$y_i = \phi_i q + \omega_i, \tag{2}$$

where $y_i \in R^{k\times1}$ is the vector of outputs at these selected sensors, $\phi_i \in R^{k\times n}$ is the matrix containing k rows of Φ corresponding to the selected k sensor locations, and $\omega_i \in R^{k\times1}$ denotes the measurement noise at these sensors. Note that ω_i is still zero-mean and Gaussian. The least square estimate of the target modal coordinates for the i-th set of selected sensor locations is

$$\hat{q}_i = (\phi_i^T\phi_i)^{-1}\phi_i^T y_i. \tag{3}$$

The sensor placement problem thus consists of choosing the best set (say, set j) of sensor locations from among the $\binom{m}{k}$ possible placements so that

$$\|\hat{q}_j - q\|^2 \le \|\hat{q}_i - q\|^2, \qquad \forall\ 1 \le i \le \binom{m}{k}. \tag{4}$$

Referring to (2), since ω_i is Gaussian and zero-mean, the probability density function is given by

$$f(y_i) = (2\pi)^{-k/2}(\det(F_{y_i}))^{-1/2}\exp(-\frac{1}{2}(y_i - m_{y_i})^T F_{y_i}^{-1}(y_i - m_{y_i})), \tag{5}$$

where $$m_{y_i} = E(y_i) = \phi_i q, \tag{6}$$

and $$F_{y_i} = E((y_i - m_{y_i})(y_i - m_{y_i})^T) = E(\omega_i\omega_i^T). \tag{7}$$

Referring to (3), it is known that the probability density function of $\hat{q}_i$ is also Gaussian, i.e.,

$$f(\hat{q}_i) = (2\pi)^{-k/2}(\det(F_{\hat{q}_i}))^{-1/2}\exp(-\frac{1}{2}(\hat{q}_i - m_{\hat{q}_i})^T F_{\hat{q}_i}^{-1}(\hat{q}_i - m_{\hat{q}_i})), \tag{8}$$

where $$m_{\hat{q}_i} = (\phi_i^T\phi_i)^{-1}\phi_i^T m_{y_i} = q, \tag{9}$$

and $$F_{\hat{q}_i} = (\phi_i^T\phi_i)^{-1}\phi_i^T F_{y_i}\phi_i(\phi_i^T\phi_i)^{-1}. \tag{10}$$

With the probability density function as in (8), it is shown in [13] that the confidence region of $\hat{q}_i$ can be described by the interior of the hyperellipsoid

$$(\hat{q}_i - q)^T F_{\hat{q}}^{-1}(\hat{q}_i - q) = c^2, \tag{11}$$

where c is a constant related to the scale of confidence. The matrix $F_{\hat{q}}^{-1}$ is also known as the Fisher Information Matrix (FIM). The volume V_i of the hyperellipsoid in (11) is given by

$$V_i = \pi^{k/2}c^k(\det(F_{\hat{q}_i}))^{1/2}(\Gamma(k/2 + 1))^{-1}, \tag{12}$$

where $\Gamma(\cdot)$ denotes the gamma function and $\det(\cdot)$ denotes the determinant. To minimize $\|\hat{q}_i - q\|^2$, the volume of the confidence region in (12) should be minimized. This is equivalent to minimizing the determinant of $F_{\hat{q}}$ since the other terms in (12) are all constants. As in [8], the measurement noise in this paper is assumed to be uncorrelated between sensors and is assumed to be of equal variance σ^2 at each sensor. Therefore, referring to (7) and (10),

$$\det(F_{\hat{q}_i}) = \det(\sigma^2(\phi_i^T\phi_i)^{-1}) = \sigma^{2k}(\det(\phi_i^T\phi_i))^{-1}. \tag{13}$$

Thus maximizing the determinant of $\phi_i^T\phi_i$ is equivalent to minimizing the determinant of $F_{\hat{q}_i}$. In the sequel, the matrix $\phi_i^T\phi_i$ will be referred to as the FIM. The GA is used in this paper to search for the best set of k sensor locations so that the determinant of the FIM is maximized. Note that a simple search is impractical due to the magnitude of $\binom{m}{k}$.

Although not investigated in this paper, it is a straightforward extension to consider measurement noise which is Gaussian and zero-mean but with arbitrary correlation matrix. Instead of applying the GA to search for the best sensor locations corresponding to the maximum determinant of $\phi_i^T\phi_i$ as discussed above, the sensor locations corresponding to the minimum determinant of $F_{\hat{q}}$ are searched for by the GA.

3. GENETIC ALGORITHM

In this section, the implementation of the GA for solving the sensor placement problem is described. Refer to [10]-[11] for more details about the GA. In the GA, there are three main operators, parent selection, crossover and mutation. The full scheme including these three operators and their modifications is now described as follows.

A. Initialization and Fitness Value

Suppose a gene pool of N chromosomes is explored in every generation of the GA. Each chromosome is encoded as a string of integers which correspond to the estimated sensor locations. Since k sensors are to be placed at k different locations among m candidate choices, each chromosome consists of k genes and each gene is an integer ranging from 1 to m. Suppose each chromosome is a set of integers $(d_1,\ldots,d_k)$, where $1 \le d_i \le m$. Let ϕ_{ij} be the matrix corresponding to the j-th chromosome in the i-th generation, then

$$\phi_{ij} = [r_{d_1}^T,\ldots,r_{d_k}^T]^T, \tag{14}$$

where r_{d_j} denotes the d_j-th row of matrix Φ. The fitness value f_{ij} associated with this chromosome is defined as

$$f_{ij} = \det(\phi_{ij}^T\phi_{ij}). \tag{15}$$

The best sensor locations estimated in generation i are defined by the chromosome corresponding to the largest fitness value

$$f_i = \max_j (f_{ij}). \tag{16}$$

The GA is used to search for the best sensor locations so that f_i converges to the maximum possible value.

B. Parent Selection

The parent selection strategy of the GA in [10]-[11] is to let the j-th chromosome in the i-th generation be selected for mating with probability

$$p_{ij} = \frac{f_{ij}}{\sum_{s=1}^{N} f_{is}} . \tag{17}$$

However, there are only a few sets of combinations of rows in Φ which result in large determinants of the FIM. After several generations, the largest fitness value tends to be much larger than that of other chromosomes. According to the parent selection scheme in (17), the best chromosome will be selected as a parent repeatedly. This causes a rapid decrease in the diversity of the gene pool, that is, the chromosomes in the succeeding generations tend to become homogeneous. Consequently, the only way of improving the fitness value in the next generation is through mutation (random changes on some genes in each chromosome).

In the sensor location problem, the fitness value is extremely sensitive to changes of the genes. For instance, the fitness values of two chromosomes can be significantly different even though they differ in only one gene. Therefore, on one hand, it is difficult to further improve the fitness value because of the homogeneity of the gene pool. On the other hand, the fitness value is usually decreased by mutation and it often requires several generations to return the fitness value back to its pre-mutation level. Drastic oscillations of the fitness value thus tend to occur once the fitness value is close to the maximum.

Since the improvement of the fitness value requires that the better chromosomes with larger fitness values be allowed to mate, we modify the GA to ensure that these chromosomes are mated in every generation. In each generation, all the best D chromosomes corresponding to the highest fitness values are allowed to survive into the next generation and serve as potential parents. These D chromosome are randomly chosen as the parents with equal probability. With this modified parent selection scheme, the problem of consistently choosing parents with larger fitness values than others is avoided. Furthermore, since the best D parents are allowed to live into the next generation, the fitness value of the next generation is greater or equal to the one in the current generation. In other words, the best fitness value in every generation is a monotonically increasing sequence. The oscillation of the fitness value at convergence is avoided and the real maximum will be eventually attained. In the next section, the effects of the modified parent selection scheme are illustrated through numerical simulation.

C. Mating

Among the D parent, pairs are randomly selected for mating. The operation of crossover is briefly illustrated as follows. Let parents 1 and 2 be chromosomes of the form

parent 1: x x x x x x x
parent 2: y y y y y y y

If the randomly assigned splice point is set at the position between the 3rd and 4th genes, then two children would be given by

child 1: x x x y y y y
child 2: y y y x x x x

Even though identical genes are not allowed to coexist when assigning a random number to each gene of the chromosome initially, it is still possible to have pairs of identical genes in the chromosome after crossover. For instance, assume that parent 1 is (1, 10, 15, 20, 30), parent 2 is (1, 2, 3, 10, 15), and the splice point for crossover is still set at the position between gene 3 and 4. One of the children would be (1, 10, 15, 10, 15). Apparently, two pairs of identical genes can occur in the child chromosomes even though no identical genes are in either of the parents. For the chromosomes with r pairs of identical genes, the effective fitness value is calculated as the determinant of $\phi'^T\phi'$ where $\phi' \in R^{(k-r)\times n}$ is the matrix corresponding to (k-r) distinct genes in the chromosome.

D. Mutation

The example above shows that it is possible to generate chromosomes with identical genes via the crossover procedure. The next theorem shows that the effective fitness value of the chromosome with pairs of identical genes is less than or equal to the fitness value of the chromosome with one of the identical genes replaced by any value different from the other genes.

Theorem 1: For every positive definite $A = \Phi^T\Phi \in R^{n\times n}$, let $r_i \in R^{1\times n}$ be the i-th row vector of Φ and $B = A - r_i^T r_i$, then

$$\det(B) = \det(A)(1 - E_{D_i}); \tag{18}$$

where $0 \le E_{D_i} \le 1$.

Proof: Refer to [14].

Replacing one of the identical genes with any value different from the other genes is analogous to exerting a forced mutation on such genes. From the above theorem, the probability of improving fitness value is increased by this forced mutation.

Along with the forced mutation, the idea of *natural mutation* is also introduced to improve the convergence of the fitness value. Natural mutation is a procedure of replacing the value of any individual gene with a random number uniformly distributed between 1 and m with probability p_m.

The algorithm is summarized as follows:

(a) Initialize N chromosomes. Set index = 0 and no-improvement tolerance L.

(b) Construct ϕ_{ij} as in (14) for each chromosome and calculate the associated fitness value f_{ij} as in (15).

(c) Select D chromosomes corresponding to the largest fitness value among N chromosomes as the parents. Pass them into the next generation.

(d) Mate D parents and generate N-D children. Invoke mutation along with the crossover procedure.

(e) The numbers contained in the genes of the best chromosome with largest fitness value are the estimated sensor locations in the current generation. If the highest fitness value in the current generation is the same as the one in the previous generation, index = index + 1; otherwise index = 0.

(f) If index > L, stop; otherwise go back to step (b) and continue. ΔΔΔ

4. NUMERICAL EXAMPLES

In this section, the GA is to be applied to select the best sensor locations for modal identification of a Space Station and a PV array. An early version of the Space Station will be considered, as shown in Figure (1), where the main truss and the PV arrays of the Space Station are illustrated. The optimal solutions obtained by the GA are compared with the suboptimal ones determined by the EfI method in [8]. The tuning parameters for the GA are set as:

N = number of chromosomes in each generation = 100;
D = number of parents selected in each generation = 30;
p_m = probability of mutation = 0.06; and
L = no-improvement tolerance = 200.

Example1: In this example, the GA is applied to select the best sensor locations for modal identification of the Space Station. For demonstration purposes, 7 fundamental FEM bending modes are selected for identification. Assume that there are 187 candidate locations and 10 sensors are to be placed on the Space Station to independently identify the target modes. The GA is to be used to search a 187×7 matrix, out of which 10 rows are to be selected so that the determinant of the corresponding FIM is a maximum. The GA is compared with the EfI method of sensor placement. Using the same example, each of the methods is applied to select the best 20, and then best 10 sensor locations for identification of the 7 target modes. The results of these two methods are compared in Table I. In both cases, the sensor locations estimated by GA correspond to larger fitness values than the sensor locations estimated by the EfI method.

Example 2: In this example, the GA is further applied to the modal identification of an individual Space Station PV array. There are thus 321 candidate sensor locations to be searched. The GA and EfI methods are also compared for this example. Each algorithm was used to select the best 30, and then best 15 sensor locations for the identification of the target modes. The results are compared in Table II. Again, in each case it is seen that the sensor locations obtained by GA correspond to larger fitness values.

5. CONCLUSION

The Genetic Algorithm with modified parent selection and children reproduction schemes has been successfully applied to the sensor placement problem for modal identification of an LSS. A feature called forced mutation is included to improve the convergence of the fitness value. By applying both the GA and the EfI method to sensor placement for an early version of the Space Station and a corresponding PV array, it has been shown that sensor configurations based on the GA are able to estimate the target mode response with higher accuracy, i.e., corresponding to a smaller confidence region.

In this paper, the performance index for the GA has been the determinant of the Fisher Information Matrix. In fact, there have been some papers using the trace or condition number of the FIM as the performance index of their proposed optimization schemes [8]-[9]. The GA can be easily modified to utilize any of these performance indices or even combinations of them with appropriately assigned weightings. The measurement noise at the sensor outputs has been assumed to be zero-mean Gaussian, uncorrelated between sensors, and constant in variance with respect to sensor location. The GA can also be easily applied to the case that the measurement noise is zero-mean Gaussian with arbitrary correlation matrix by using the original definition of the Fisher Information matrix given in (10).

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	10 sensors		20 sensors	
	largest fitness value	# of sensor locations different	largest fitness value	# of sensor locations different
GA2	1.2652e4	1	8.6236e5	1
EfI	1.2594e4		7.8711e5	

Table I. Comparison of simulation results by the GA and EfI method for example 1.

	15 sensors		30 sensors	
	largest fitness value	# of sensor locations different	largest fitness value	# of sensor locations different
GA2	8.7122e5	2	1.0465e8	3
EfI	8.6024e5		1.0363e8	

Table II. Comparison of simulation results by the GA and EfI method for example 2.